Modelling Recursion in Event-B

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Universität Düsseldorf

Rodin User and Developer Workshop
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Development and Verification of Recursive Programs

- We seek a method:
  - Easy to apply (also to larger programs)
  - Tool support (such as Rodin)
  - Data refinement
  - Imperative programs and procedures
- Some related work:[1]
  - Program Verification (Hoare)
  - Proof Outlines (Owicki/Gries)
  - Refinement Calculus (Morgan)
  - Refinement Calculus (Back)

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[1] Missing from the list: B-Method and VDM, for instance

Method of Program Development

- A model describes an "*abstract program*"
- "*Concrete programs*" are an indirect target
  - Maybe not reached by refinement
- "*Abstract program*":
  - only call-by-reference parameters
  - and no global variables

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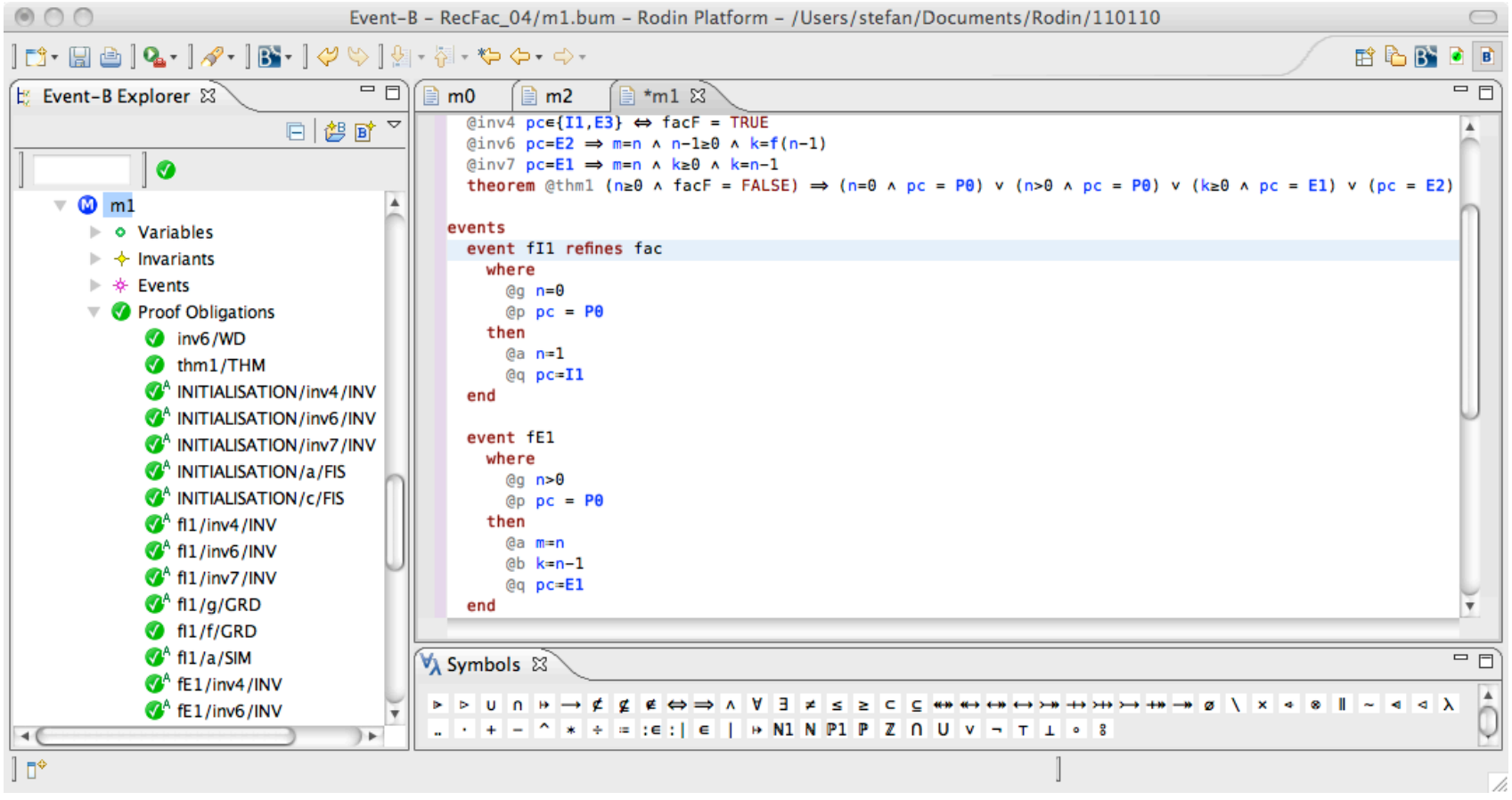
Use of Event-B (and Rodin)



- Clear and direct mapping to Event-B machines
- Use **Rodin** to do the proofs
- Keep **Event-B** notion of refinement
- For now write the "mapped" model directly in Event-B

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Proof Outline of Factorial Procedure

Program Verification

$\{\, n \geq 0 \wedge n = N \,\}$ *Fac*(n) $\{\, n = N! \,\}$ ::

$\{\, n \geq 0 \wedge n = N \,\}$

if $n = 0$ then

$\{\, n = 0 \wedge n = N \,\}$

$n := 1$

$\{\, n = N! \,\}$

else

$\{\, n > 0 \wedge n = N \,\}$

var $m := n$;

$\{\, m = N \wedge n > 0 \wedge n = N{-}1 \,\}$

$n := n{-}1$;

$\{\, m = N \wedge n \geq 0 \wedge n = (N{-}1) \,\}$

Fac(n);

$\{\, m = N \wedge n = (N{-}1)! \,\}$

$n := m * n$

$\{\, n = N! \,\}$

end

$\{\, n = N! \,\}$

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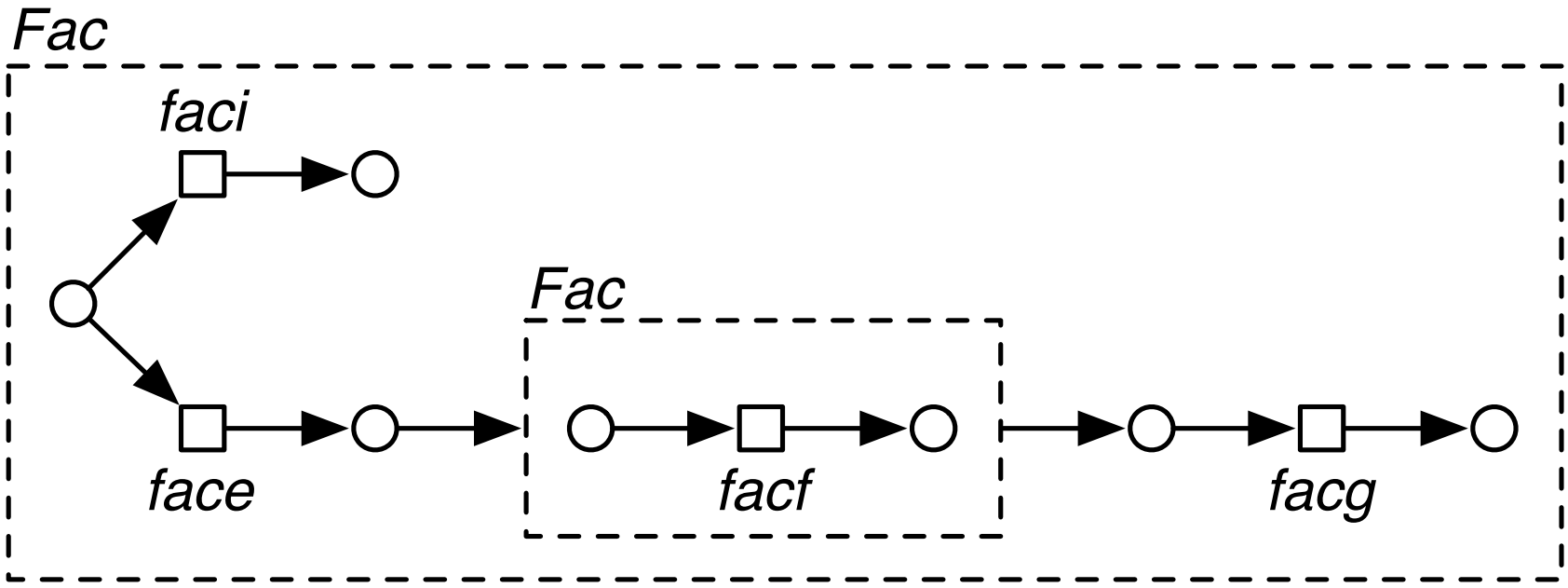
Graphical Notation for a Proof Outline Using Events

Towards Event-B



- *faci* $=$ when $n = 0$ then $n := 1$ end
- *face* $=$ when $n > 0$ then $m, n := n, n - 1$ end
- *facf* $=$ when $n \geq 0$ then $n := n!$ end
- *facg* $=$ $n := m * n$

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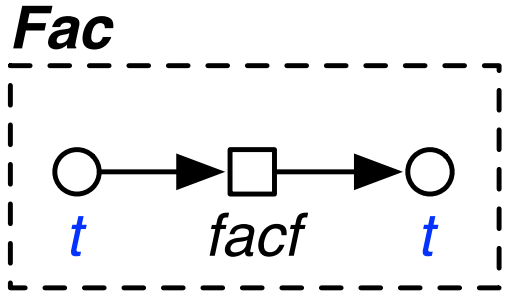
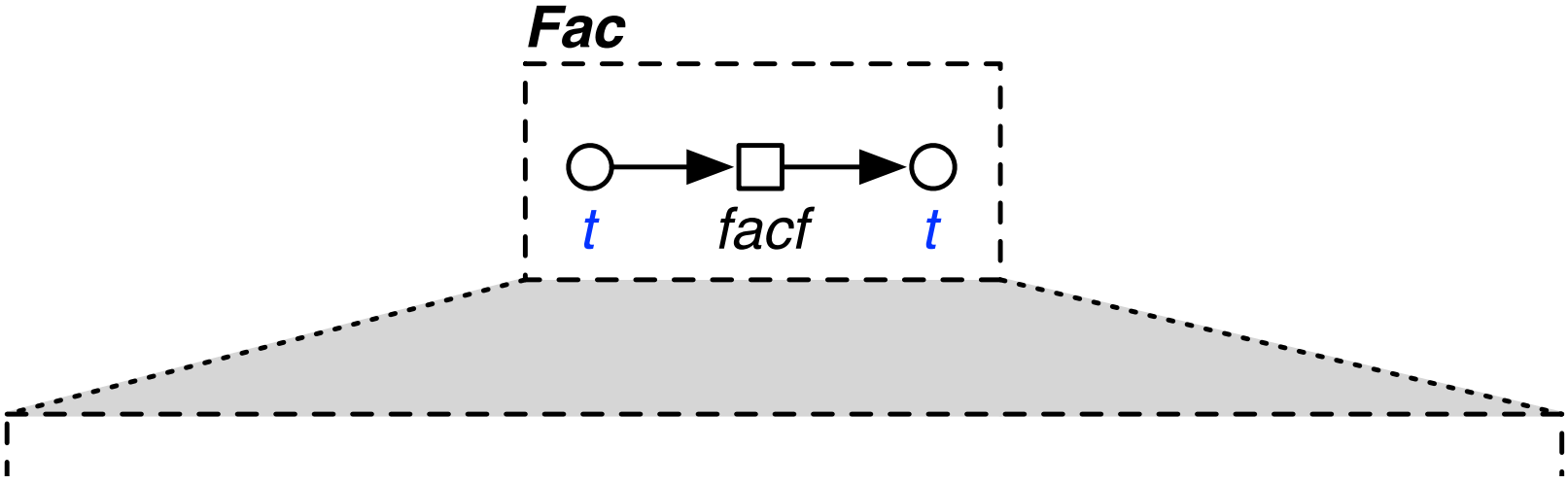
Factorial Specification



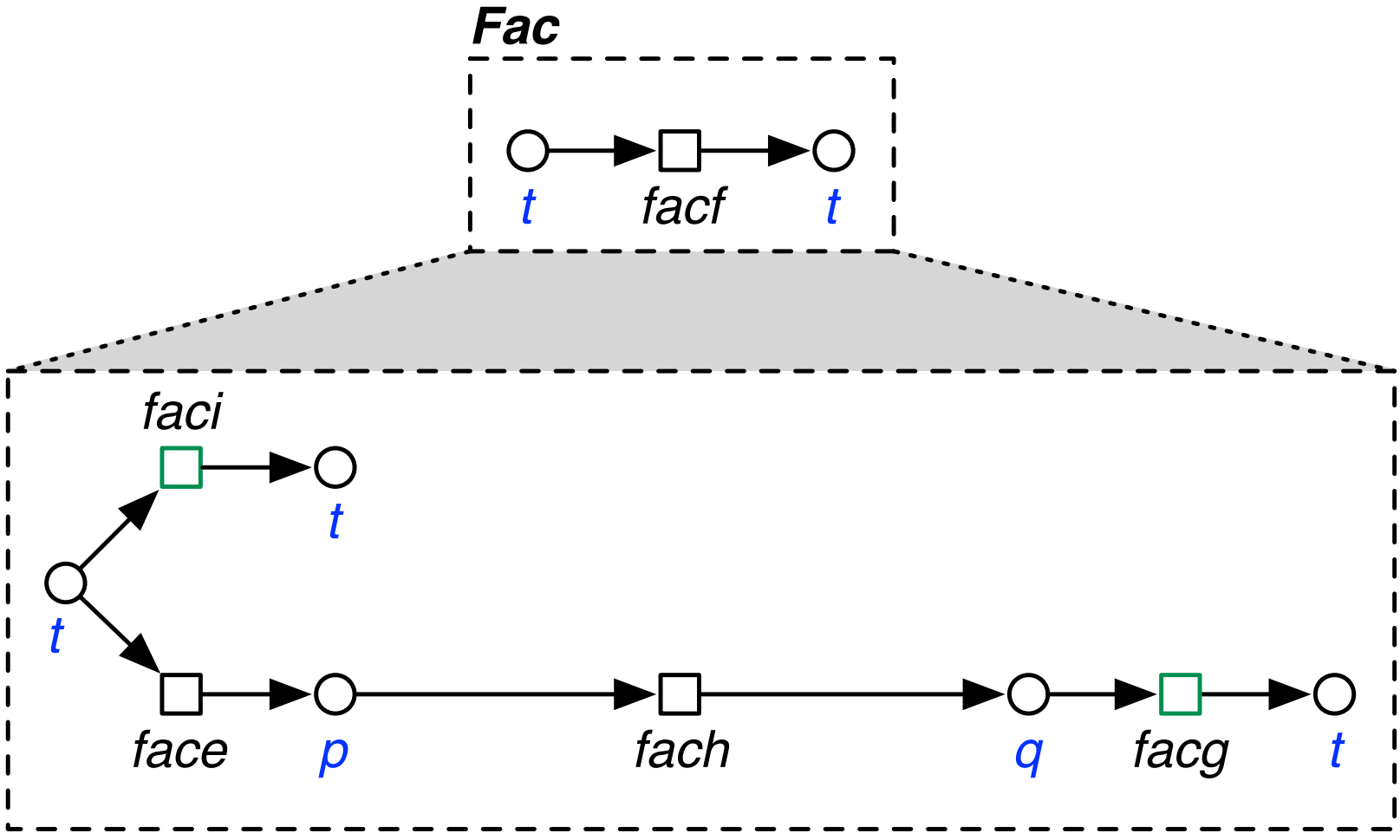
@t *true*

facf $=$ when $n \geq 0$ then $n := n!$ end

Factorial Refinement Sketch



Factorial Refinement Sketch



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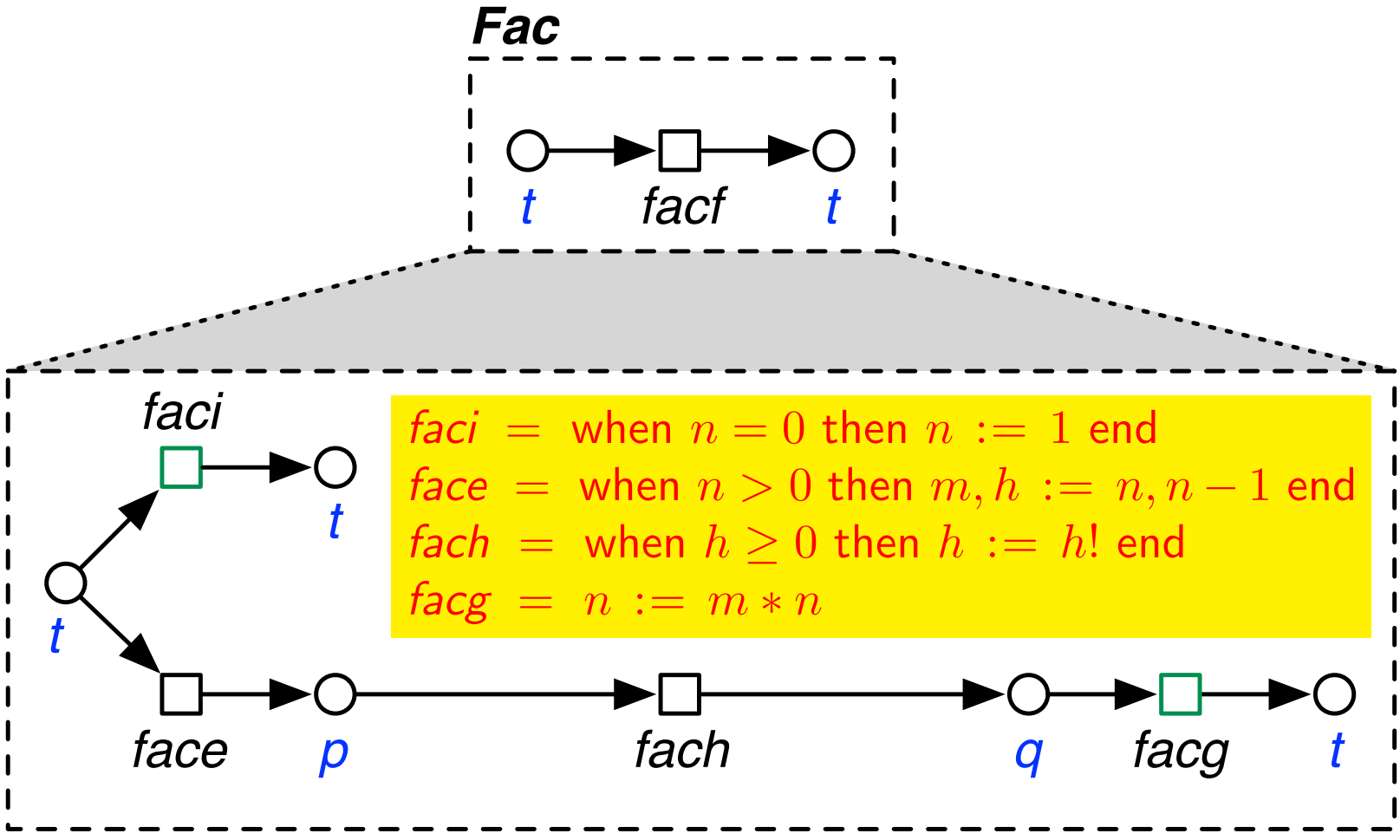

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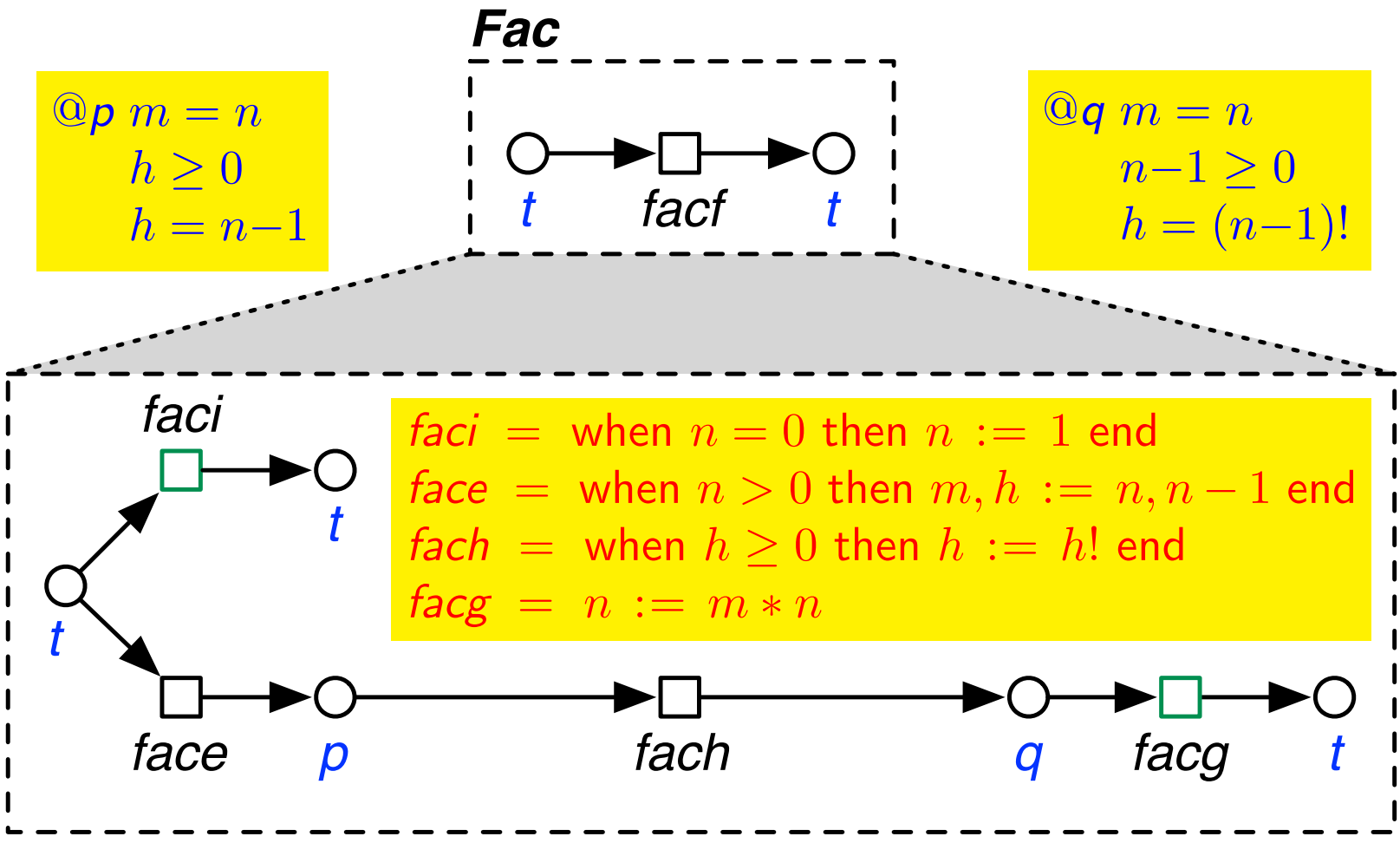

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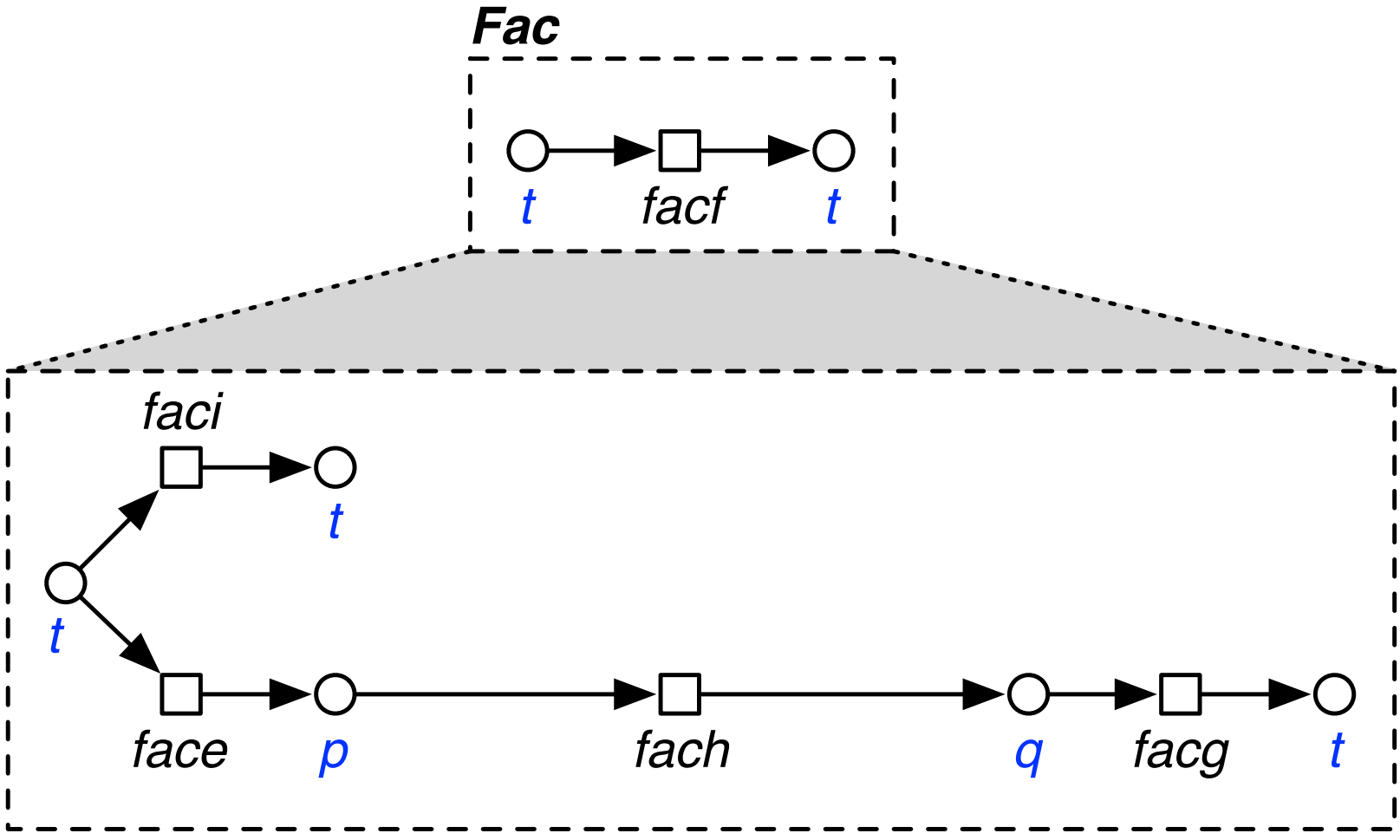
Factorial Refinement Sketch



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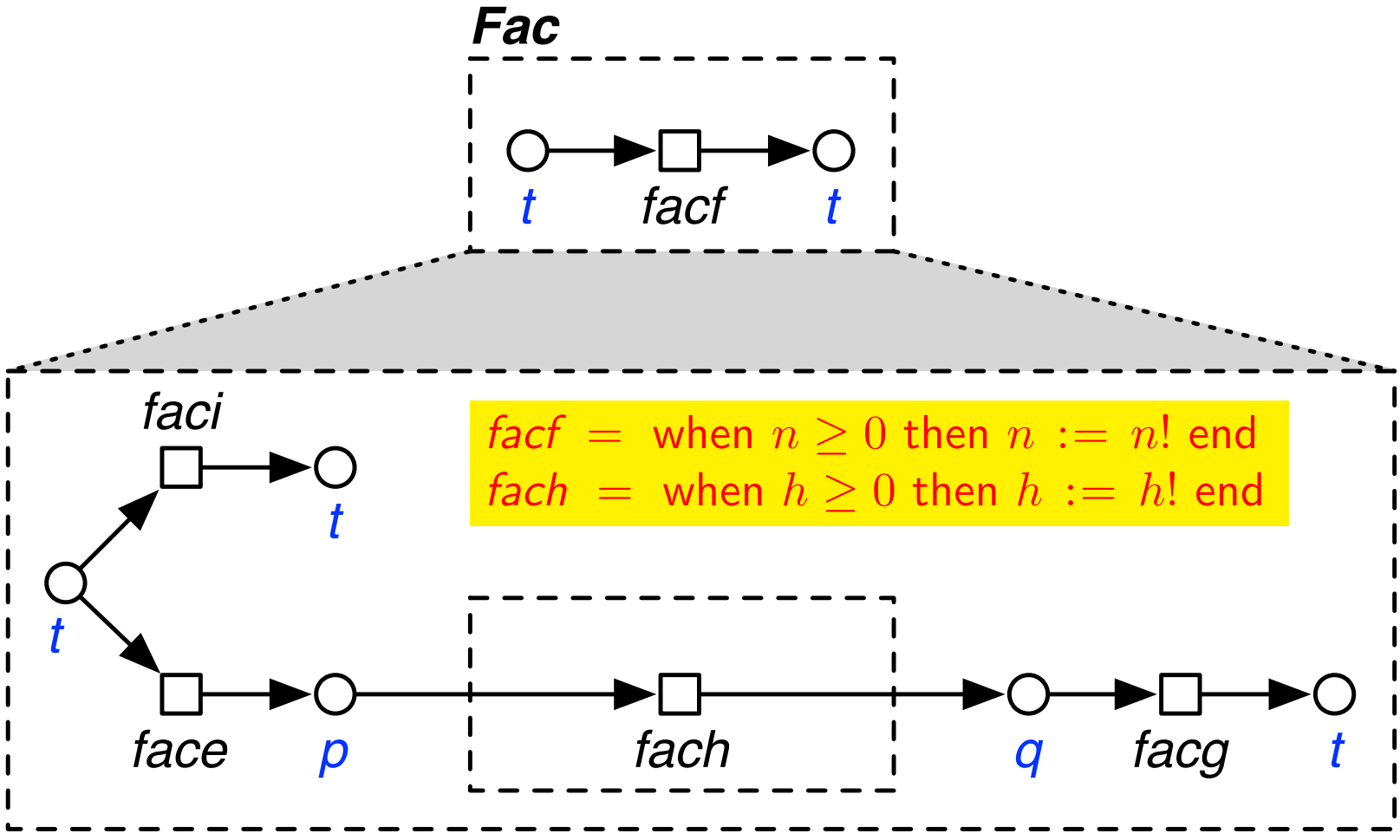
A Recursive Reference?



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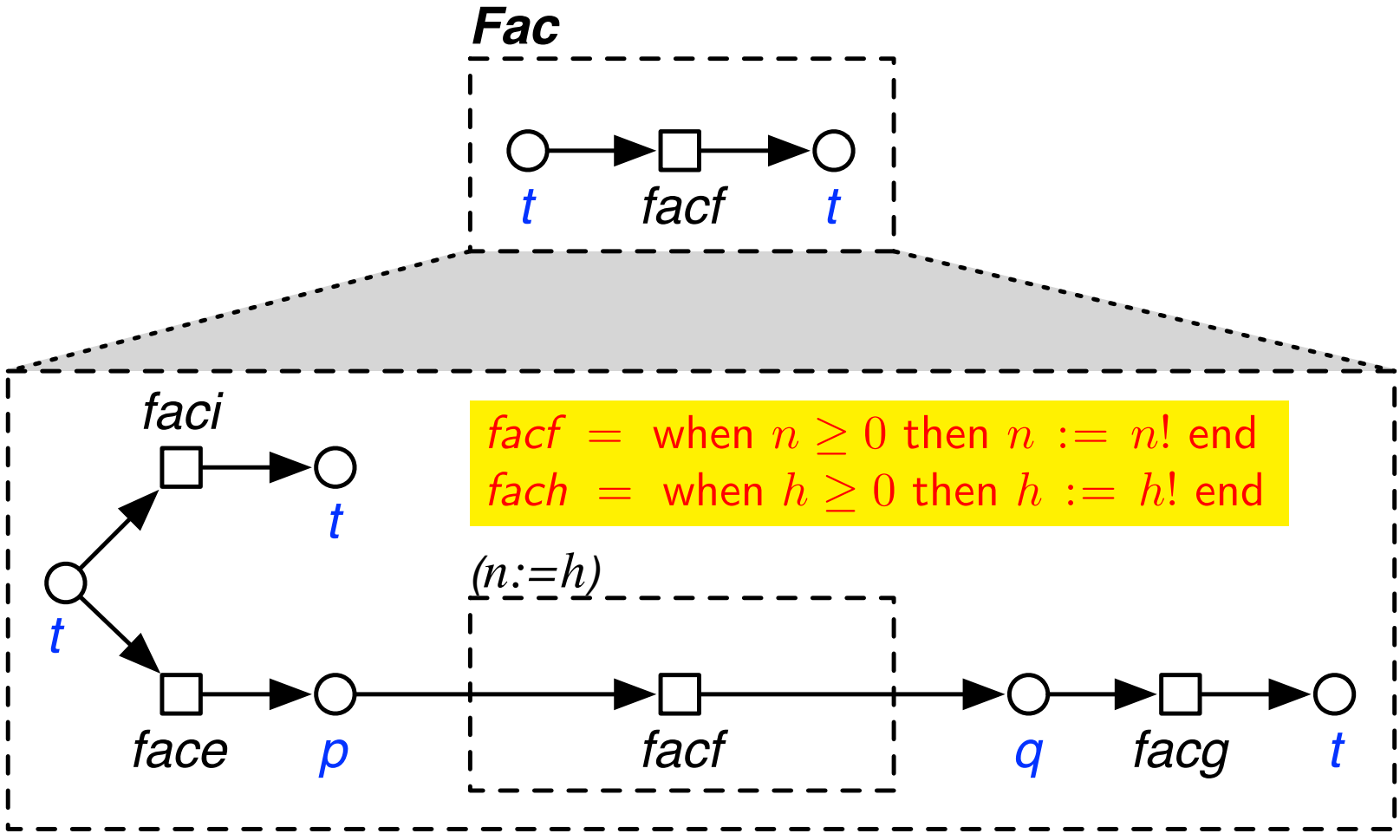

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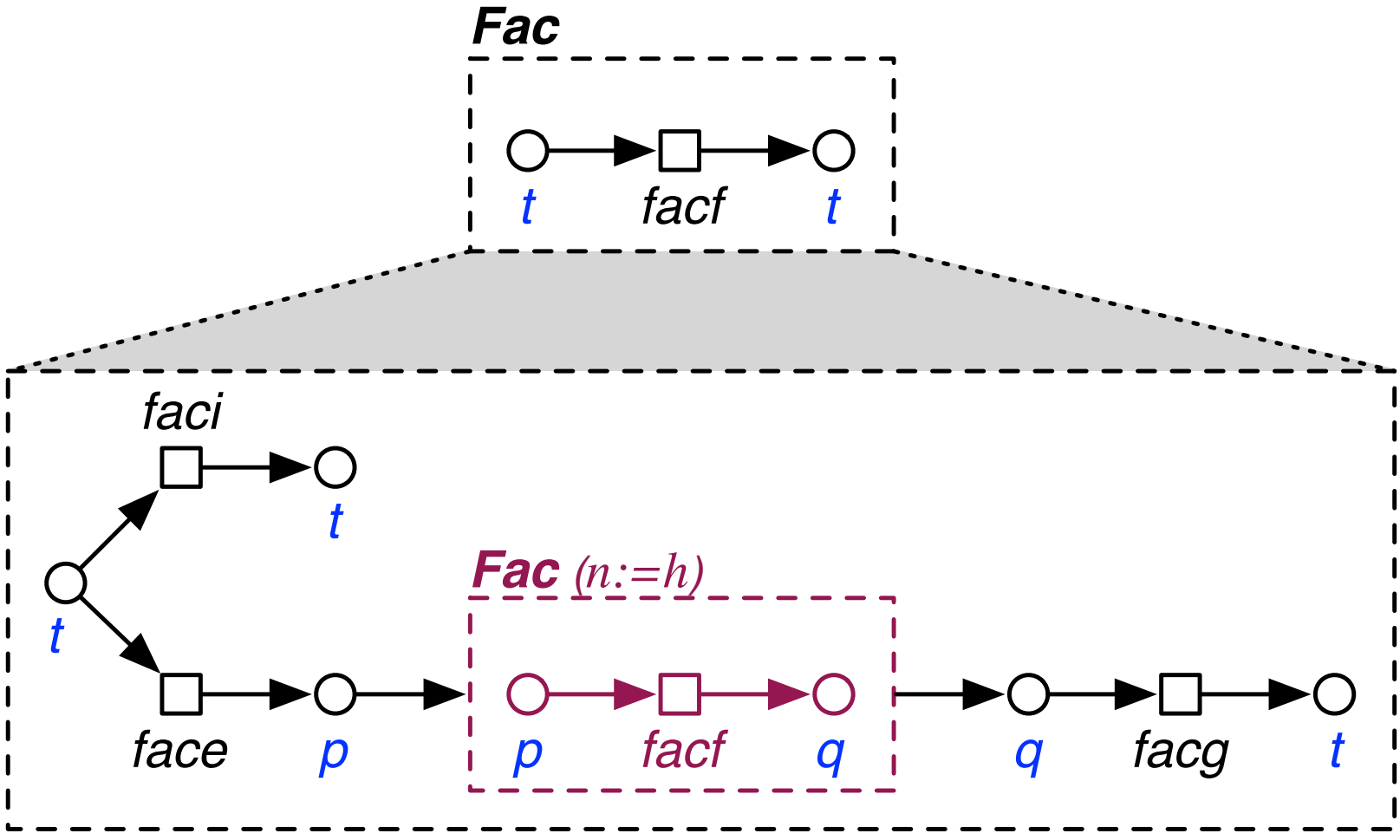
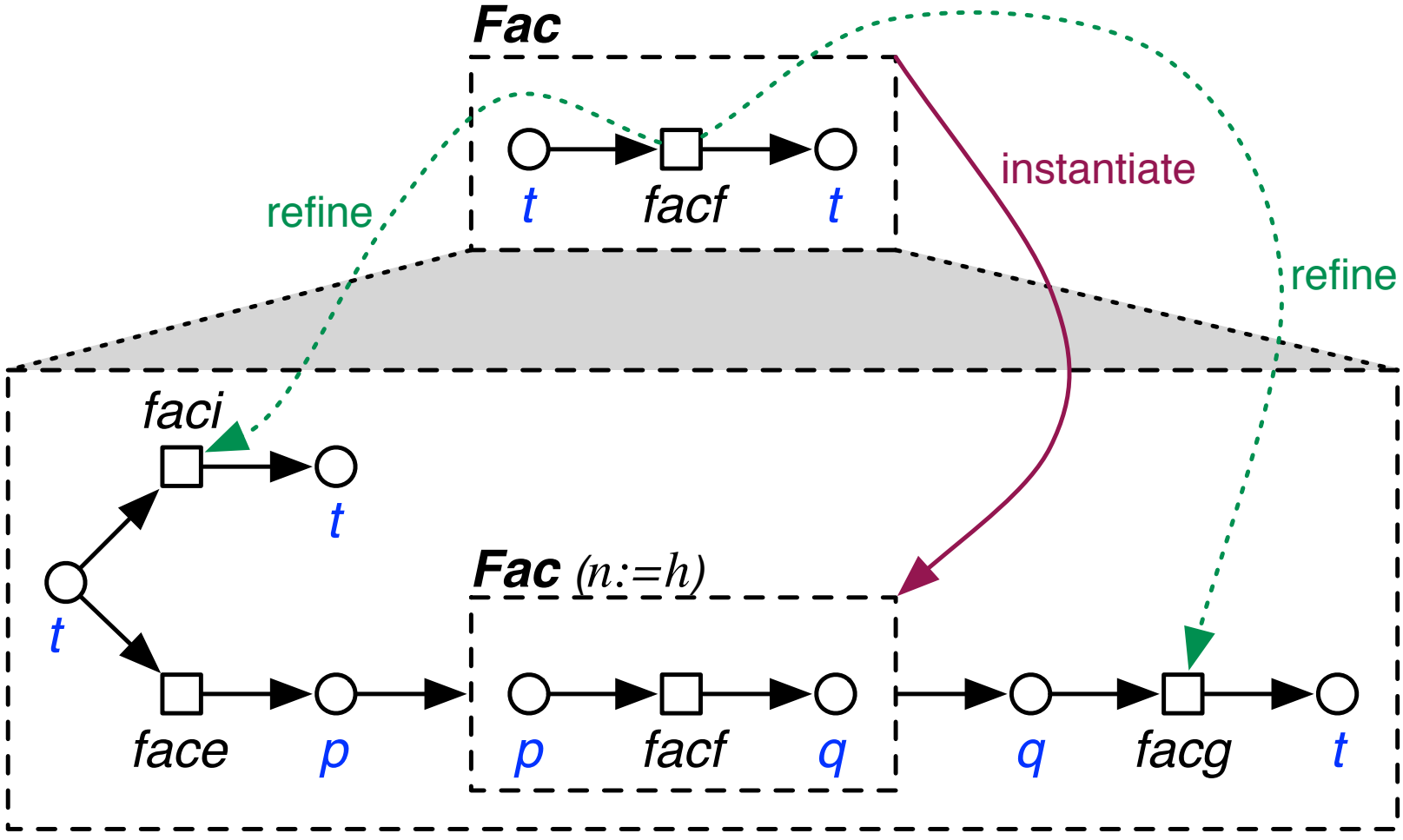
Recursion by Instantiation



Completed Factorial Model



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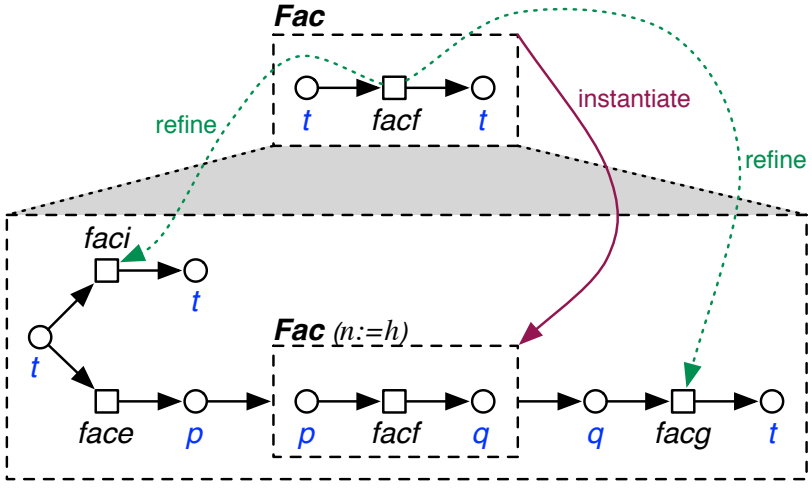
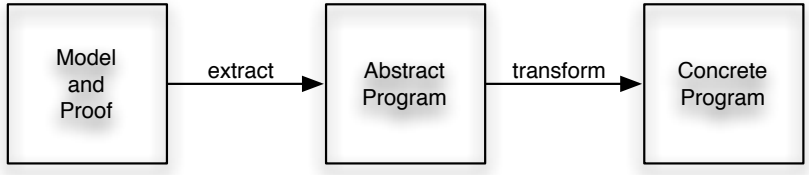
Extracted Factorial Program





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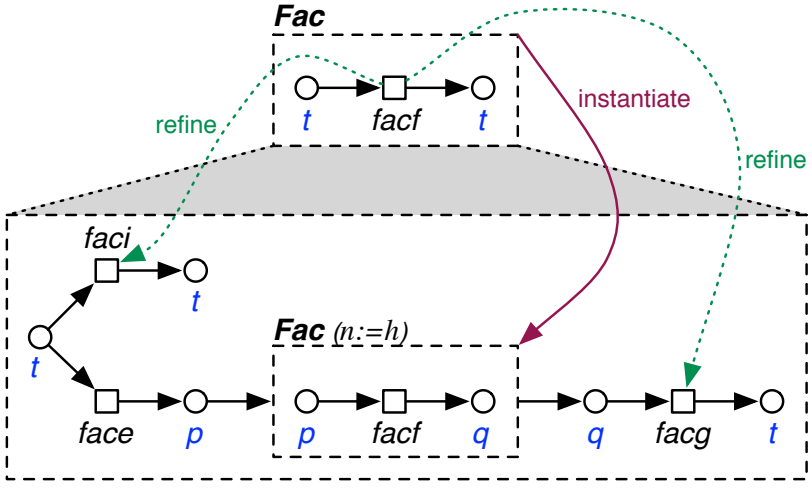

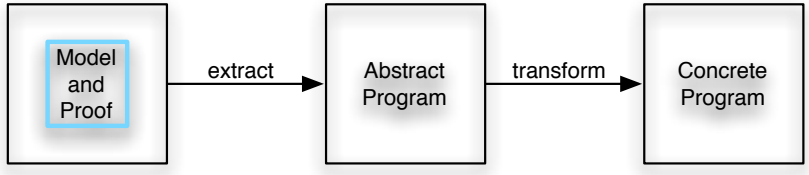

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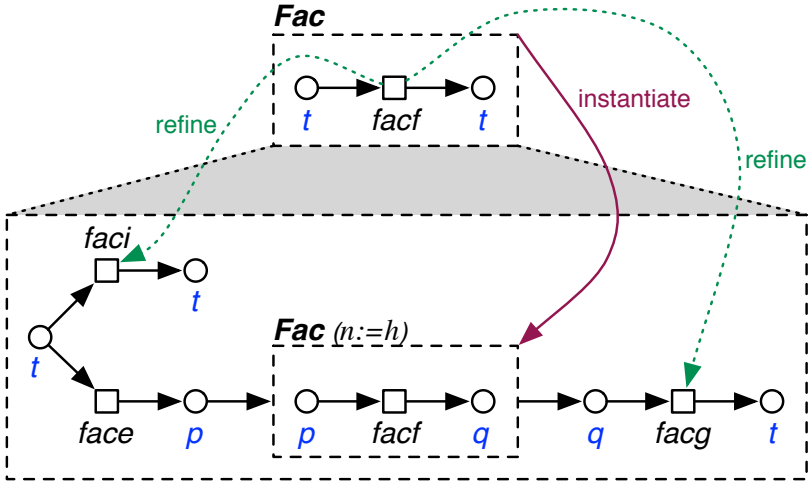

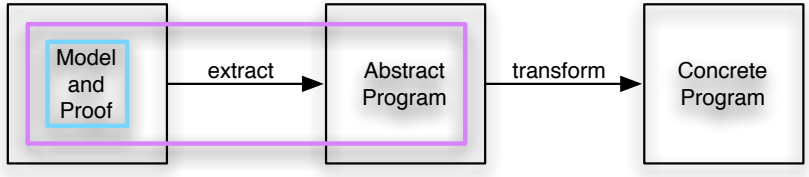

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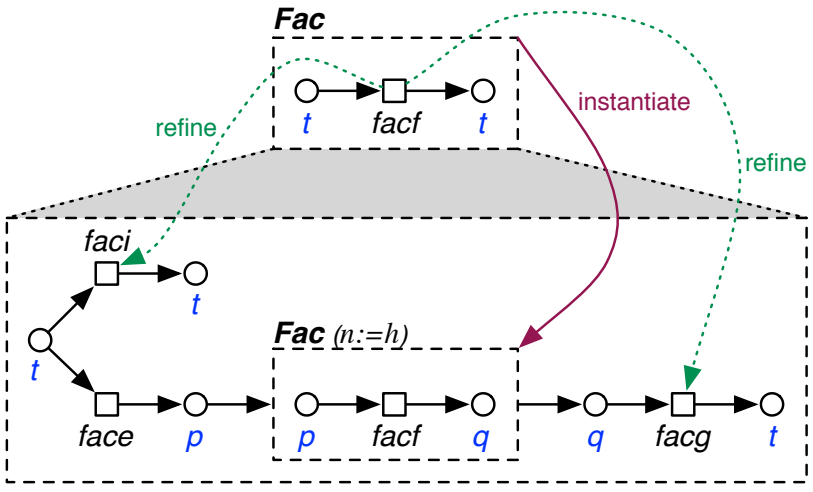

$$
\begin{array}{l}
\mathit{Fac}(n) \ :: \\
\quad \text{if } n = 0 \text{ then} \\
\qquad n \ := \ 1 \\
\quad \text{else} \\
\qquad \text{var } m,\, h \ := \ n,\, n{-}1\,; \\
\qquad \mathit{Fac}(h)\,; \\
\qquad n \ := \ m * h \\
\quad \text{end}
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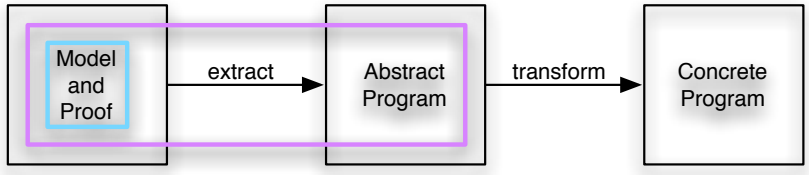

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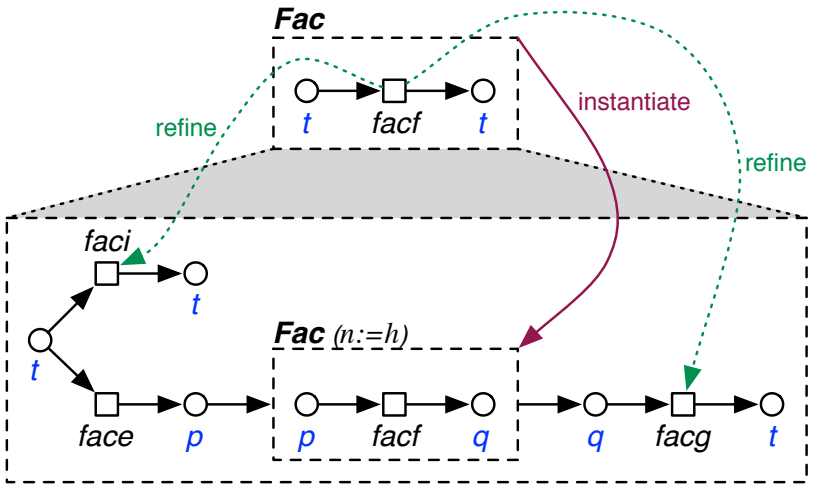

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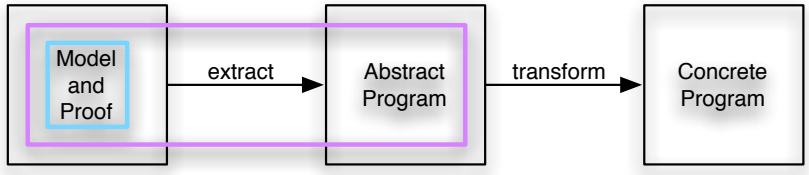

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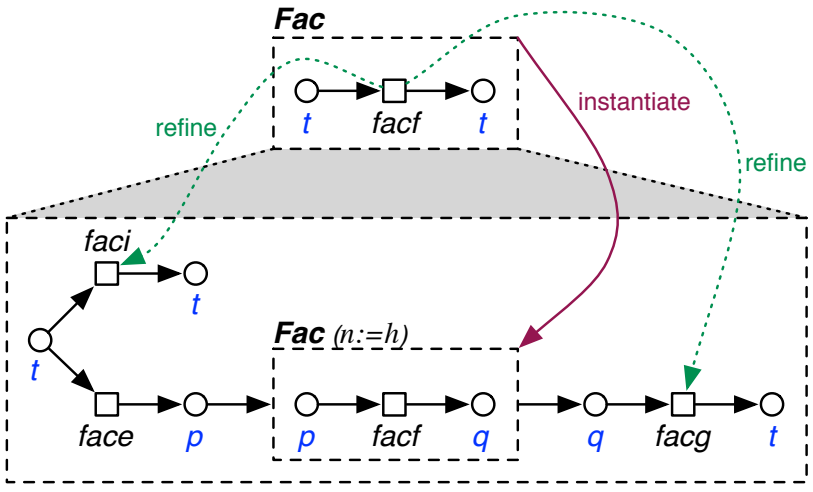

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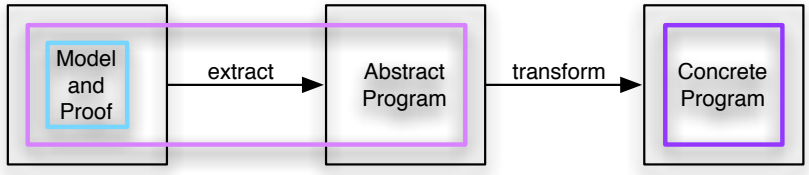

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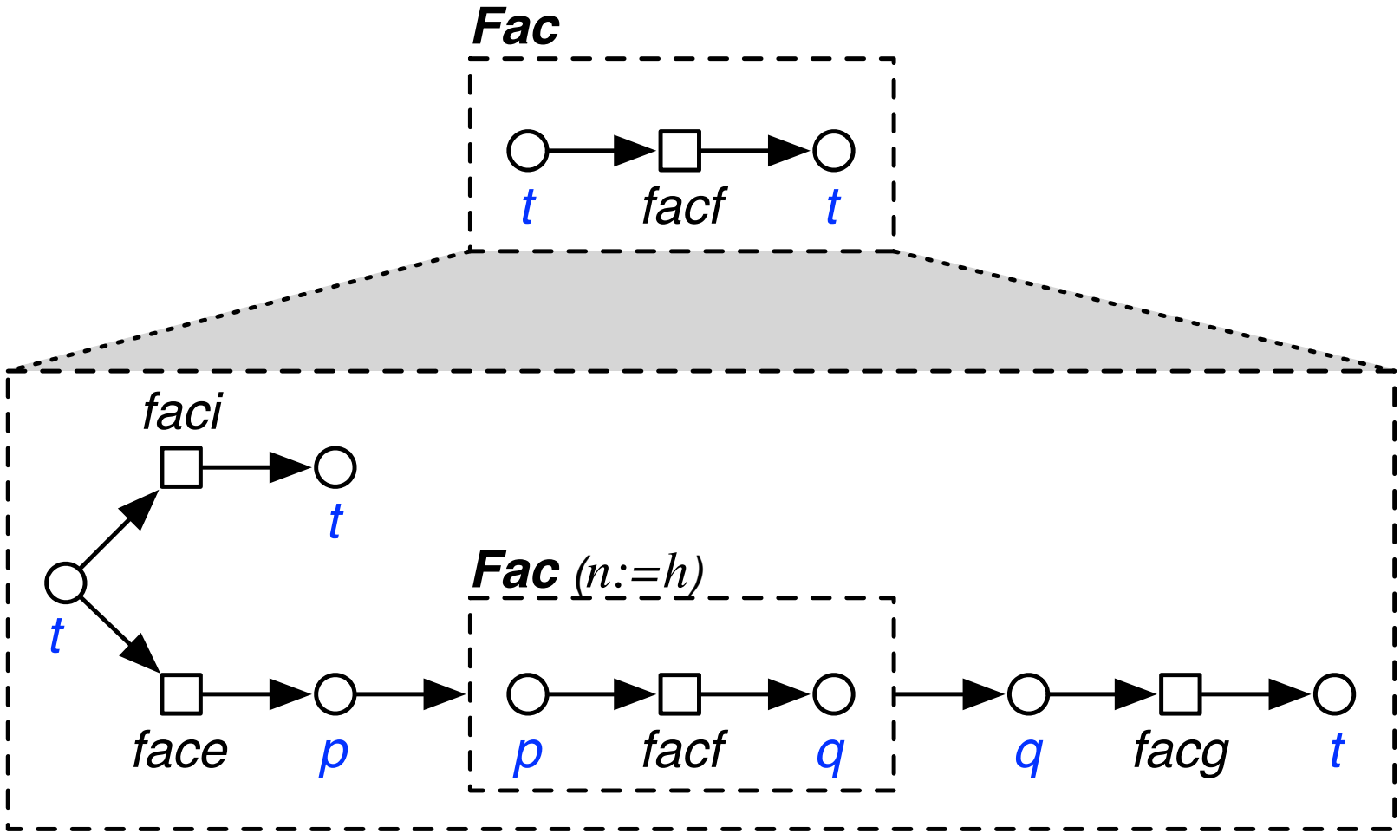
Towards a Concrete Program



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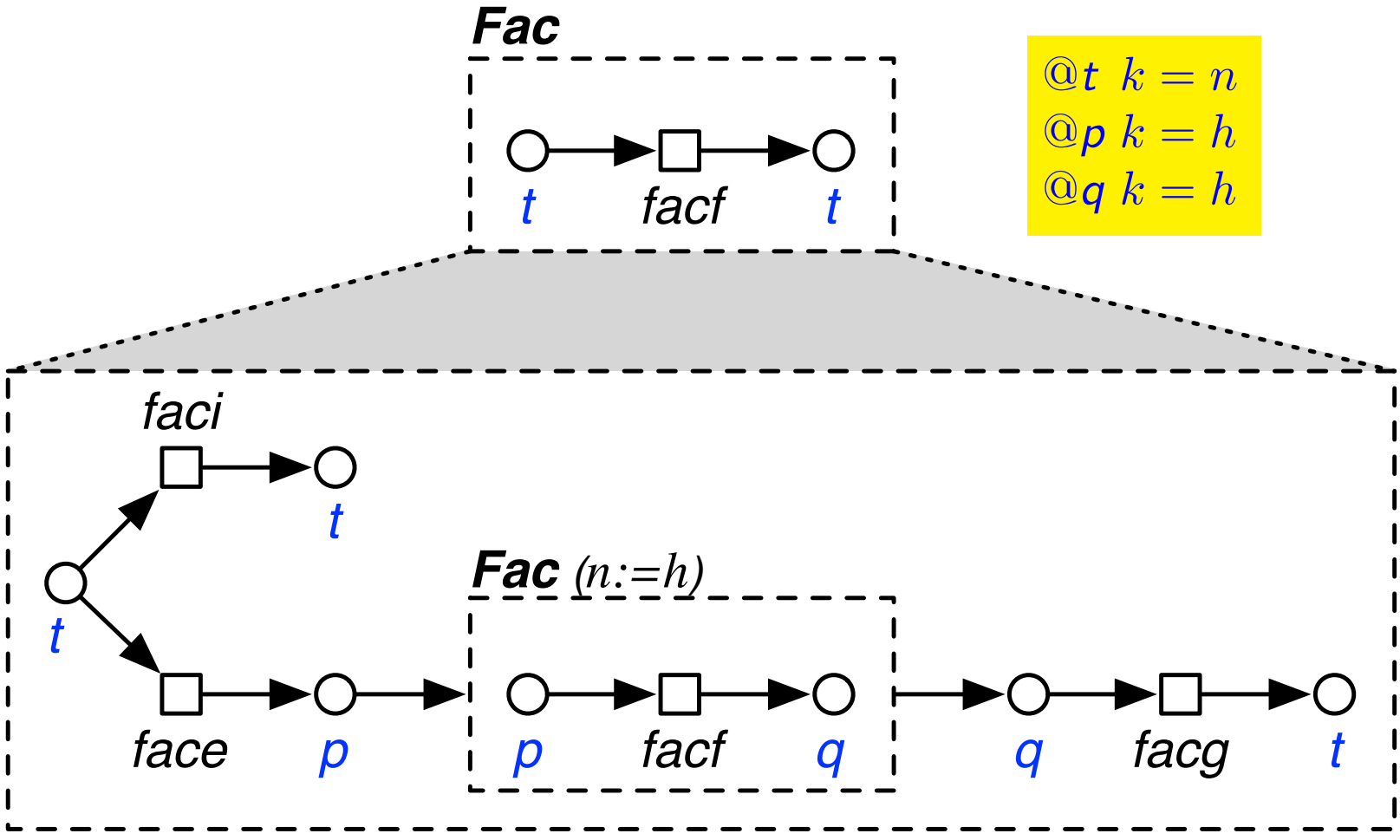

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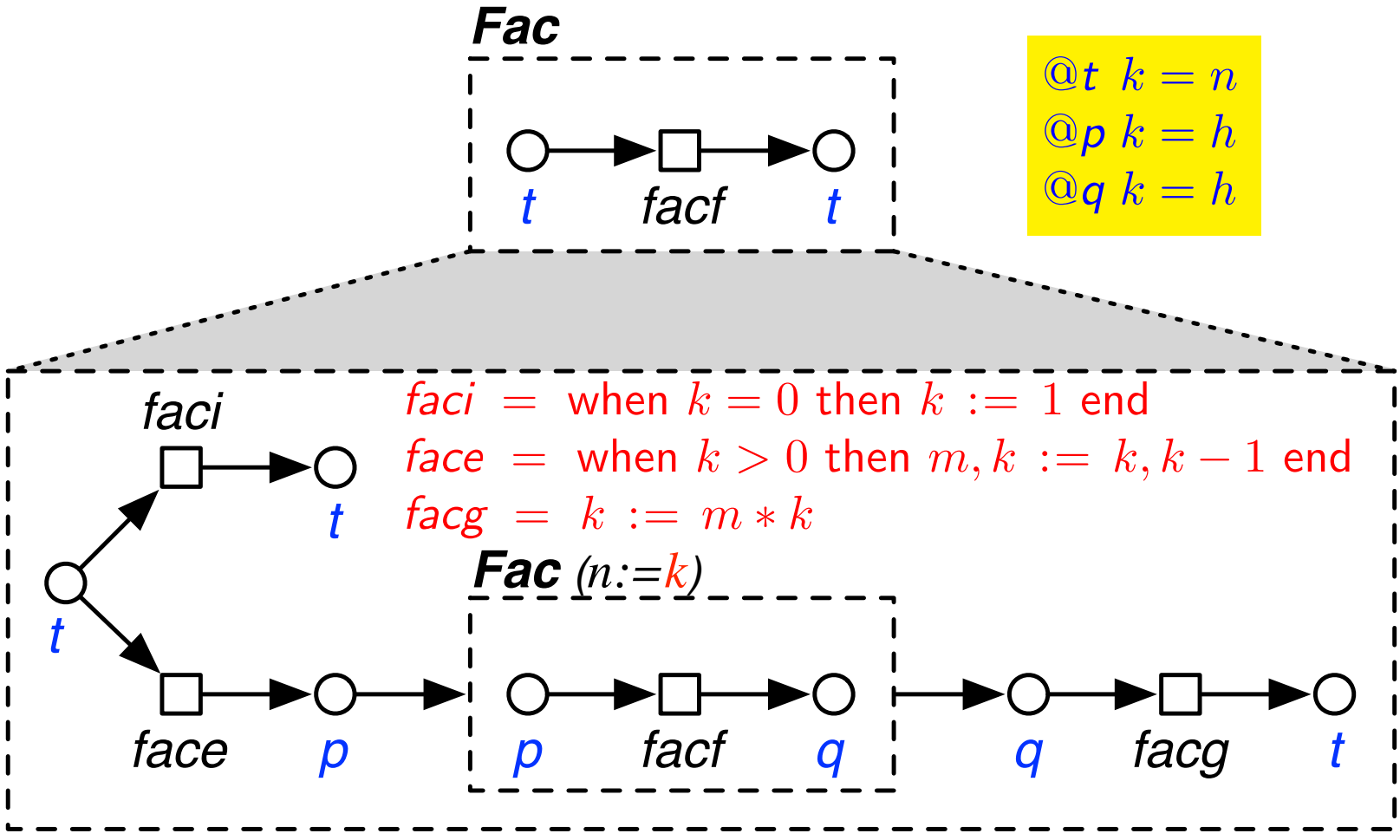

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Concrete Factorial Program

```
Fac(k) ::
  if k = 0 then
    k := 1
  else
    var m := k;
    k := k−1;
    Fac(k);
    k := m ∗ k
  end
```

Concrete Factorial Program

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transform $\longrightarrow$

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global var k
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    Fac;
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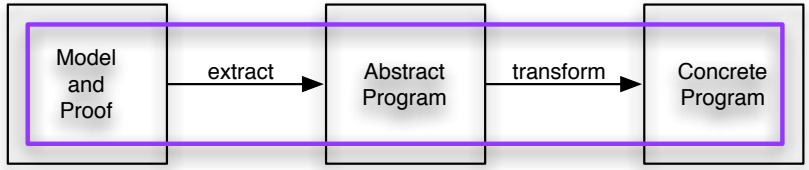

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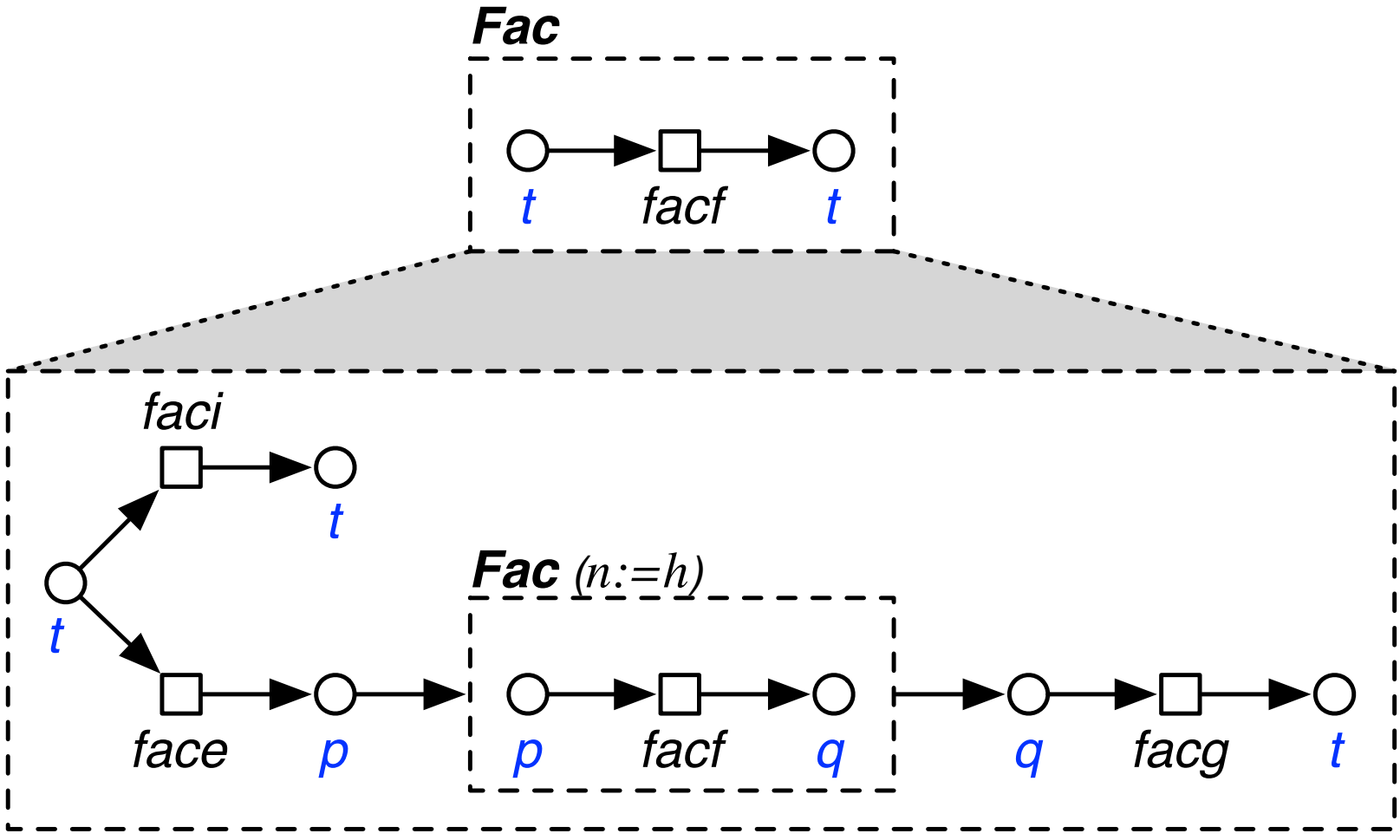
Termination of the Factorial Procedure



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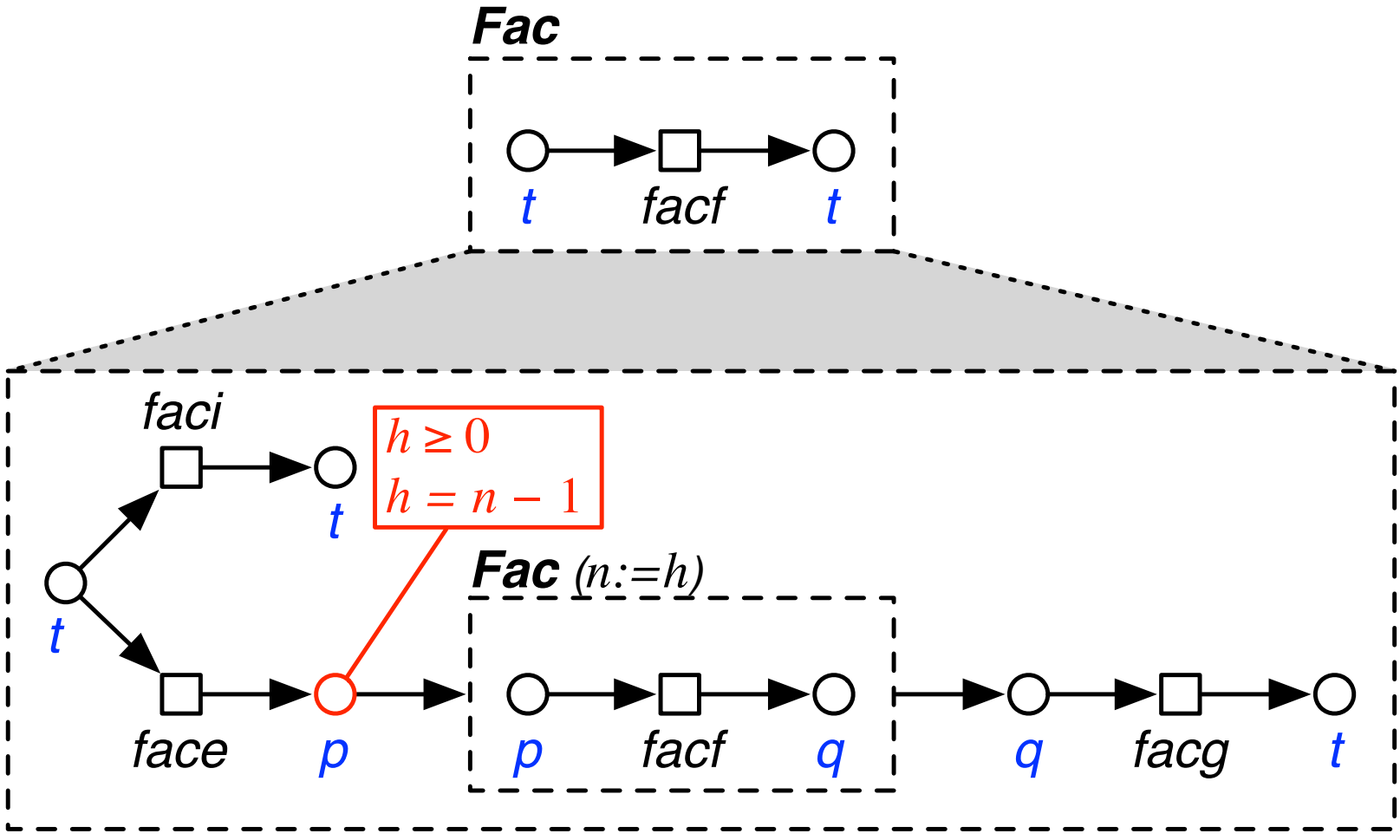

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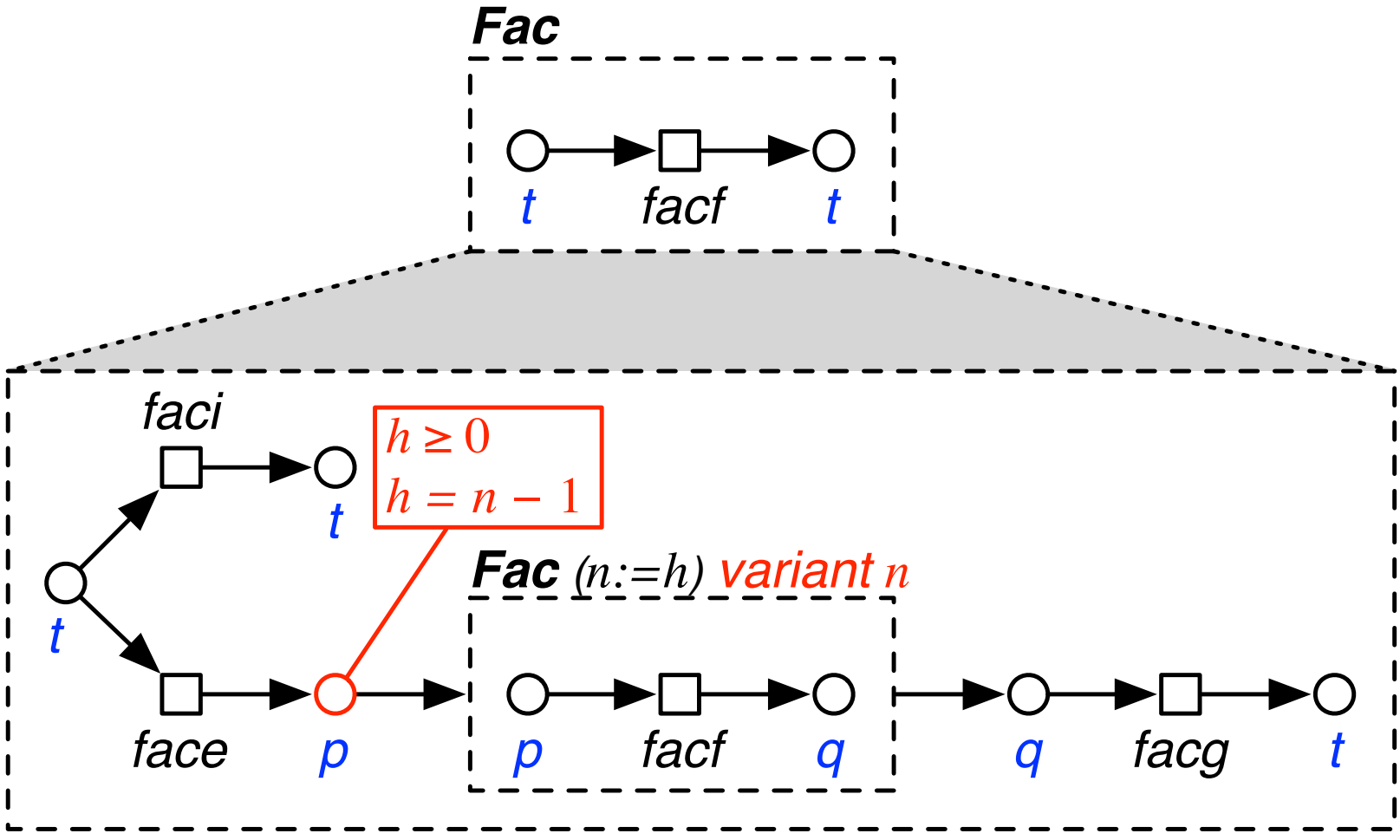
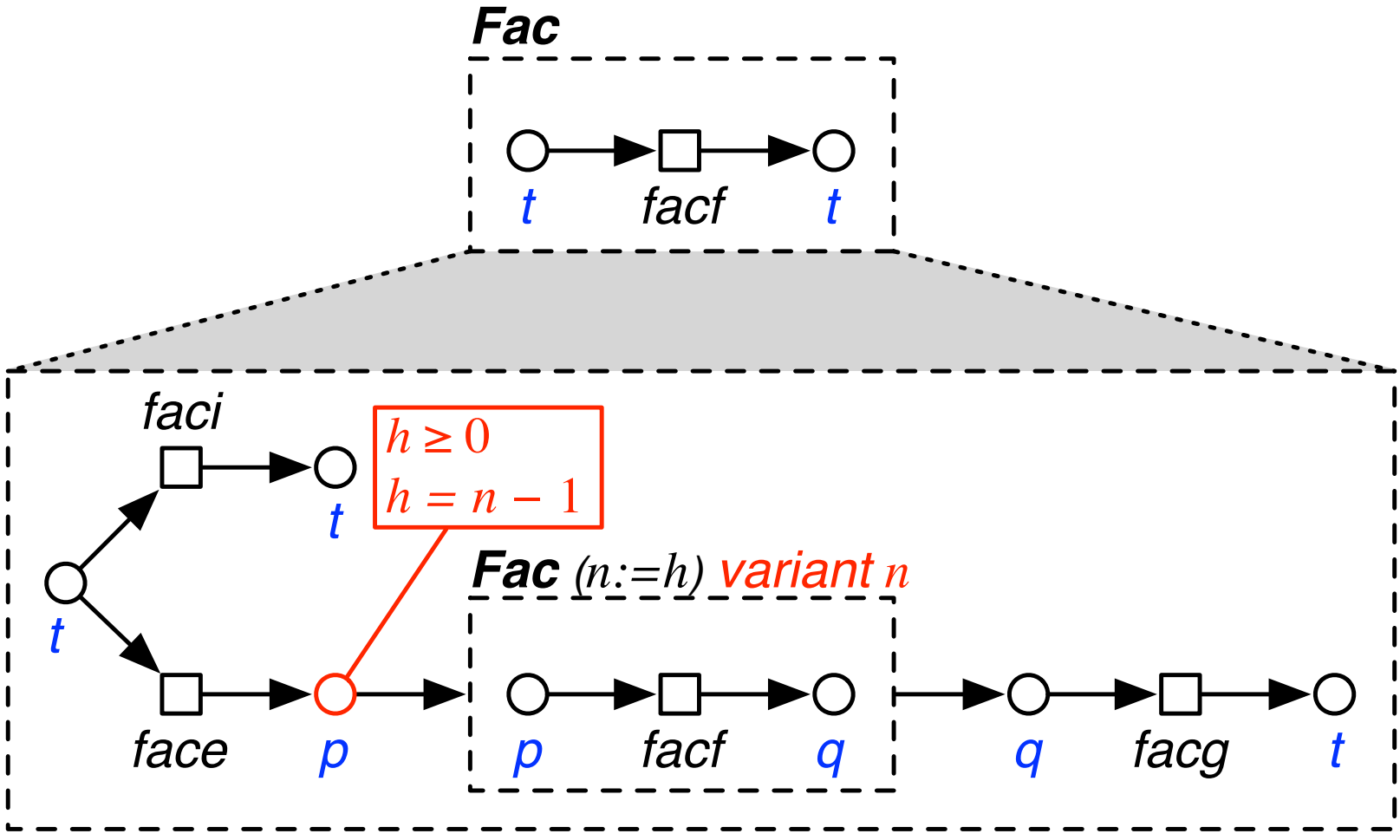


Termination of the Factorial Procedure



To be proved: $n \geq 0 \wedge n - 1 < n$

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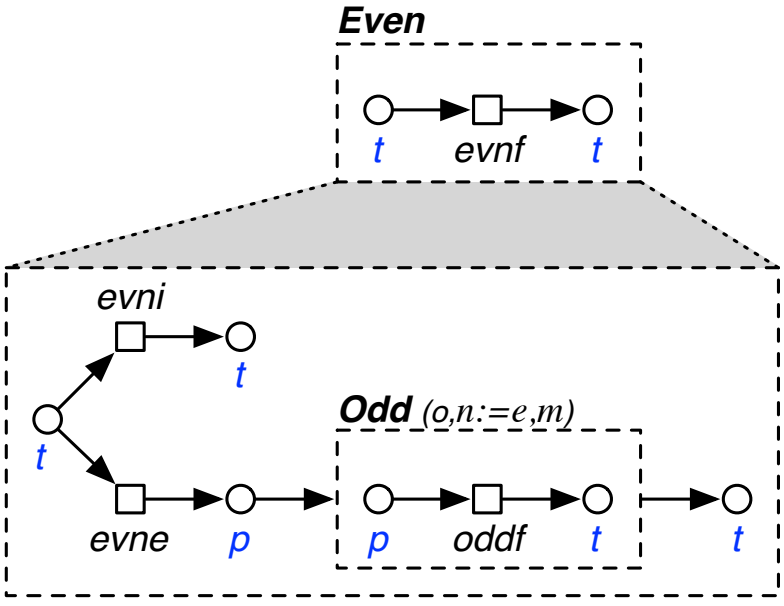
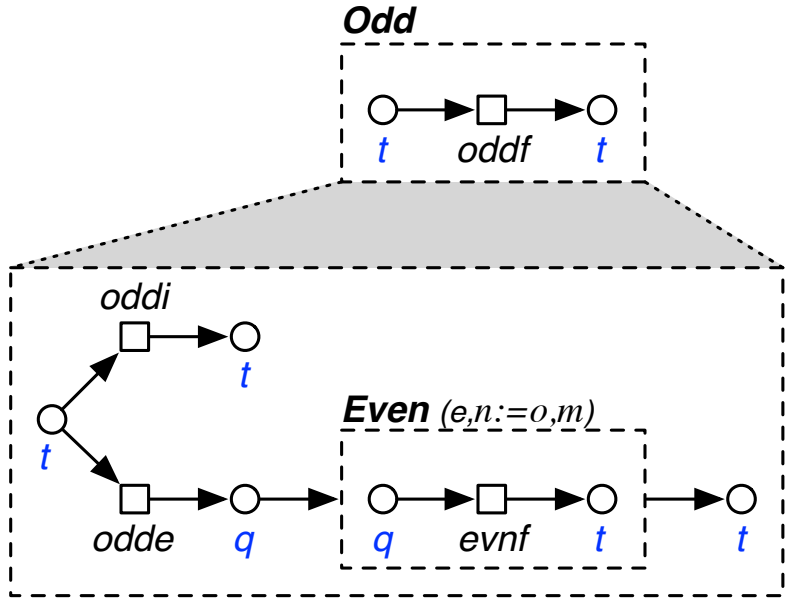
Specification of EvenOdd

"Defunctionalisation" (J. C. Reynolds)





evnf = when $n \geq 0$ then $e :=$ "$n \bmod 2 = 0$" end

evni = when $n = 0$ then $e :=$ TRUE end
evne = when $n > 0$ then $m := n-1$ end

@*p* $m = n-1$

oddf = when $n \geq 0$ then $o :=$ "$n \bmod 2 \neq 0$" end

oddi = when $n = 0$ then $o :=$ FALSE end
odde = when $n > 0$ then $m := n-1$ end

@*q* $m = n-1$

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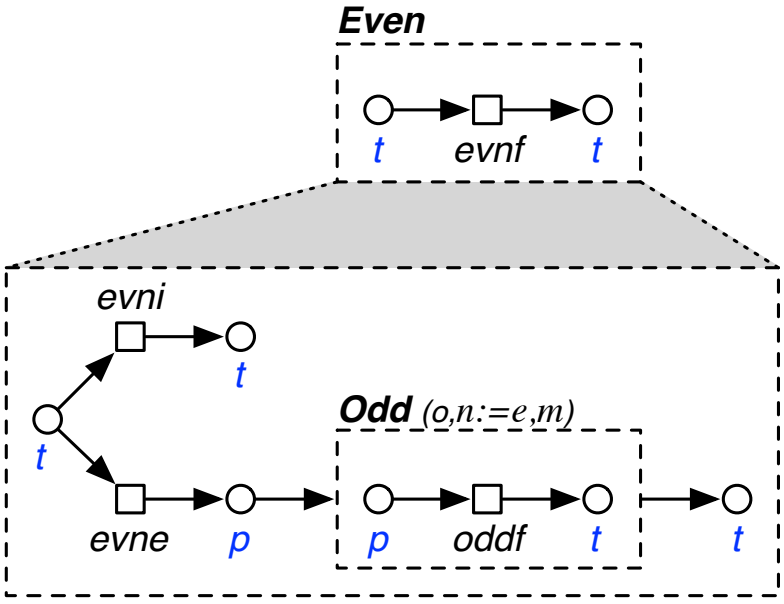
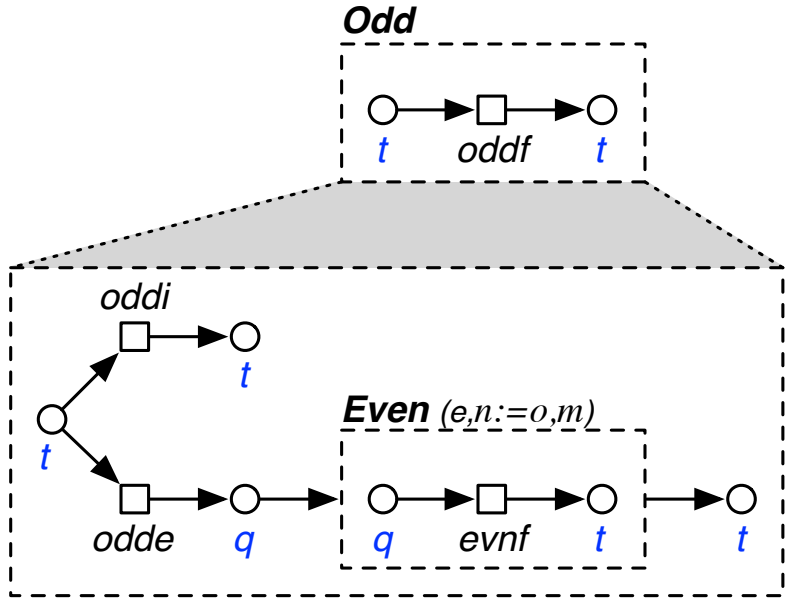
Specification of EvenOdd

"Defunctionalisation" (J. C. Reynolds)





Even(e, n) ::
  if $n = 0$ then
    e := TRUE
  else
    var m := $n-1$; *Odd*(e, m)
  end

Odd(o, n) ::
  if $n = 0$ then
    o := FALSE
  else
    var m := $n-1$; *Even*(o, m)
  end

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Conclusion

- Method for sequential program development
  - Recursion
  - Mutual Recursion
  - Termination
- To be investigated:
  - Modularity
  - Concurrency
  - Soundness