

THEORY

Seq // A theory of sequences defined as finite partial functions.

TYPE PARAMETERS

A

OPERATORS

•**seq** : seq($a : \mathbb{P}(A)$) **EXPRESSION PREFIX** // The sequence operator
direct definition
seq($a : \mathbb{P}(A)$) $\triangleq \{n, f \cdot n \in N \wedge f \in 1..n \rightarrow a \mid f\}$ // a set of finite total functions

•**seqSize** : seqSize($s : \text{seq}(A)$) **EXPRESSION PREFIX** // size of sequences
direct definition
seqSize($s : \text{seq}(A)$) $\triangleq \text{card}(s)$

•**seqIsEmpty** : seqIsEmpty($s : \text{seq}(A)$) **PREDICATE PREFIX** // predicate operator that checks whether a given sequence is empty
direct definition
seqIsEmpty($s : \text{seq}(A)$) $\triangleq \text{seqSize}(s) = 0$

•**seqHead** : seqHead($s : \text{seq}(A)$) **EXPRESSION PREFIX** // the head of a non-empty sequence
well-definedness condition
 $\neg \text{seqIsEmpty}(s)$
direct definition
seqHead($s : \text{seq}(A)$) $\triangleq s(1)$

•**seqTail** : seqTail($s : \text{seq}(A)$) **EXPRESSION PREFIX** // the tail of a non-empty sequence
well-definedness condition
 $\neg \text{seqIsEmpty}(s)$
direct definition
seqTail($s : \text{seq}(A)$) $\triangleq \lambda i \cdot i \in 1.. \text{seqSize}(s)-1 \mid s(i+1)$

•**seqPrepend** : seqPrepend($s : \text{seq}(A)$, $e : A$) **EXPRESSION PREFIX** // prepends an element to a sequence
direct definition
seqPrepend($s : \text{seq}(A)$, $e : A$) $\triangleq \{1 \mapsto e\} \cup (\lambda i \cdot i \in 2.. \text{seqSize}(s)+1 \mid s(i-1))$

•**seqAppend** : seqAppend($s : \text{seq}(A)$, $e : A$) **EXPRESSION PREFIX** // appends an element to a sequence
direct definition
seqAppend($s : \text{seq}(A)$, $e : A$) $\triangleq s \cup \{(\text{seqSize}(s)+1) \mapsto e\}$

THEOREMS

seqsIsFinite : $\forall s, a \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow \text{finite}(s)$

seqsMonotone : $\forall s, a, b \cdot (a \subseteq A \wedge s \in \text{seq}(a) \wedge a \subseteq b \Rightarrow s \in \text{seq}(b))$

tailSeqIsSeq : $\forall s, a \cdot a \subseteq A \wedge s \in \text{seq}(a) \wedge \neg \text{seqIsEmpty}(s) \Rightarrow \text{seqTail}(s) \in \text{seq}(a)$

seqPrependIsSeq : $\forall s, a, e \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow (\text{seqPrepend}(s, e) \in \text{seq}(a \cup \{e\}))$

seqAppendIsSeq : $\forall s, a, e \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow (\text{seqAppend}(s, e) \in \text{seq}(a \cup \{e\}))$

THEOREMS

seqsIsFinite : $\forall s, a \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow \text{finite}(s)$
seqsMonotone : $\forall s, a, b \cdot (a \subseteq A \wedge s \in \text{seq}(a) \wedge a \subseteq b \Rightarrow s \in \text{seq}(b))$
tailSeqIsSeq : $\forall s, a \cdot a \subseteq A \wedge s \in \text{seq}(a) \wedge \neg \text{seqIsEmpty}(s) \Rightarrow \text{seqTail}(s) \in \text{seq}(a)$
seqPrependIsSeq : $\forall s, a, e \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow (\text{seqPrepend}(s, e) \in \text{seq}(a \cup \{e\}))$
seqAppendIsSeq : $\forall s, a, e \cdot a \subseteq A \wedge s \in \text{seq}(a) \Rightarrow (\text{seqAppend}(s, e) \in \text{seq}(a \cup \{e\}))$

PROOF RULES

sequences Rules :

Metavariables

- $s \in \mathbb{Z} \leftrightarrow A$
- $a \in \mathbb{P}(A)$

Inference Rules

•**seqIsFiniteInf** : (interactive) sequences are finite
▪ $s \in \text{seq}(a)$

- $\text{finite}(s)$

•**tailSeqIsSeqInf** : (automatic) tail is a sequence

- $s \in \text{seq}(a)$
- $\neg \text{seqIsEmpty}(s)$

- $\text{seqTail}(s) \in \text{seq}(a)$

END