Theory Plug-in User Manual

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The Theory plug-in is a contribution to the Rodin platform that facilitates the **specification**, **validation**, **deployment** and **use** of language and proof extensions for the Event-B methodology. Language extensions are additions to the Event-B mathematical language in the form of 1) datatypes, 2) operators and 3) axiomatic definitions. Proof extensions are additions to the Event-B proving infrastructure in the form of rewrite rules, inference rules and polymorphic theorems. The specification of extensions is achieved by means of *theories* Ⓣ. The validation of extensions is achieved by means of proof obligations whenever appropriate. This user manual provides a comprehensive overview of the plug-in's functionality and capabilities.

For a quick start guide, the user can skip to Section 3.

ⓘ The Event-B **mathematical language** refers to the language used to write axioms, invariants, guards etc. in Event-B models.

ⓘ Event-B theories are Rodin file just like contexts and machines.

1 Motivation

Work on the Theory plug-in started as an effort to create a *Rule-based Prover* for Event-B much like the ML prover in Atelier-B. The Rule-based Prover, as it was known then, supported the definition, validation and use of rewrite rules. The Theory plug-in is the successor of the Rule-based Prover, and offers much more functionality.

Prior to Rodin v2.0, the mathematical language used in Event-B was fixed. As such, it was not possible to define reusable polymorphic operators. A workaround was to define any required operators as set constructs in contexts. Originally, contexts were supposed to provide a parametrization of machines. The aforementioned limitations of the Event-B language lead to users to use contexts for purposes for which they were not intentionally devised. Examples of operators that can be useful to users include the sequence operator (which was present in classical B mathematical language) and the bag operator.

In Rodin v2.0, a dynamic parser has been implemented for the Event-B AST. The Theory plug-in was a natural candidate for defining and using mathematical extensions. To provide a comprehensive platform, cover for a wider range of proof rules was also needed.

ML is a rule-based prover as opposed to the semi-decision procedure PP.

2 Capabilities

The Theory plug-in has the following capabilities:

1. ***Theory Definition:***
   (a) Definition of **operators**: operators can be defined as predicate or expression operators. An expression operator is an operator that "returns" an expression, an example existing operator is *card*. A predicate operator is one that "returns" a predicate, an example existing predicate operator is $finite$.
   (b) Definition of **datatypes**: datatypes are defined by supplying the types on which they are polymorphic, a set of constructors one of which has to be a base constructor. Each constructor may or may not have destructors.
   (c) Definition of **axiomatic definitions**: axiomatic definitions are defined by supplying the types, a set of operators, and a set of axioms.
   (d) Definition of **rewrite rules**: rewrite rules are one-directional equalities that can be applied from left to right. The Theory plug-in can be used to define rewrite rules.
   (e) Definition of **inference rules**: inference rules can be used to infer new hypotheses, split a goal into sub-goals or discharge sequents.
   (f) Definition of **polymorphic theorems**: theorems can be defined and validated once, and can then be imported into sequents of proof obligations if a suitable type instantiation is available.
   (g) **Validation of extensions**: where appropriate, proof obligations are generated to ensure soundness of extensions. This includes, proof obligations for validity of inference and rewrite rules, as well as proof obligations to validate operator properties such as associativity and commutativity.
2. ***Theory Deployment:*** this step signifies that a theory is ready for use. Theories can be deployed after they have been optionally validated by the user. It is strongly advisable to discharge all proof obligations before deployment.

Once a theory has been deployed to its designated project, all its extensions (mathematical and proof extensions) can be used in models. In later sections, we show the two different scopes of theory availability.

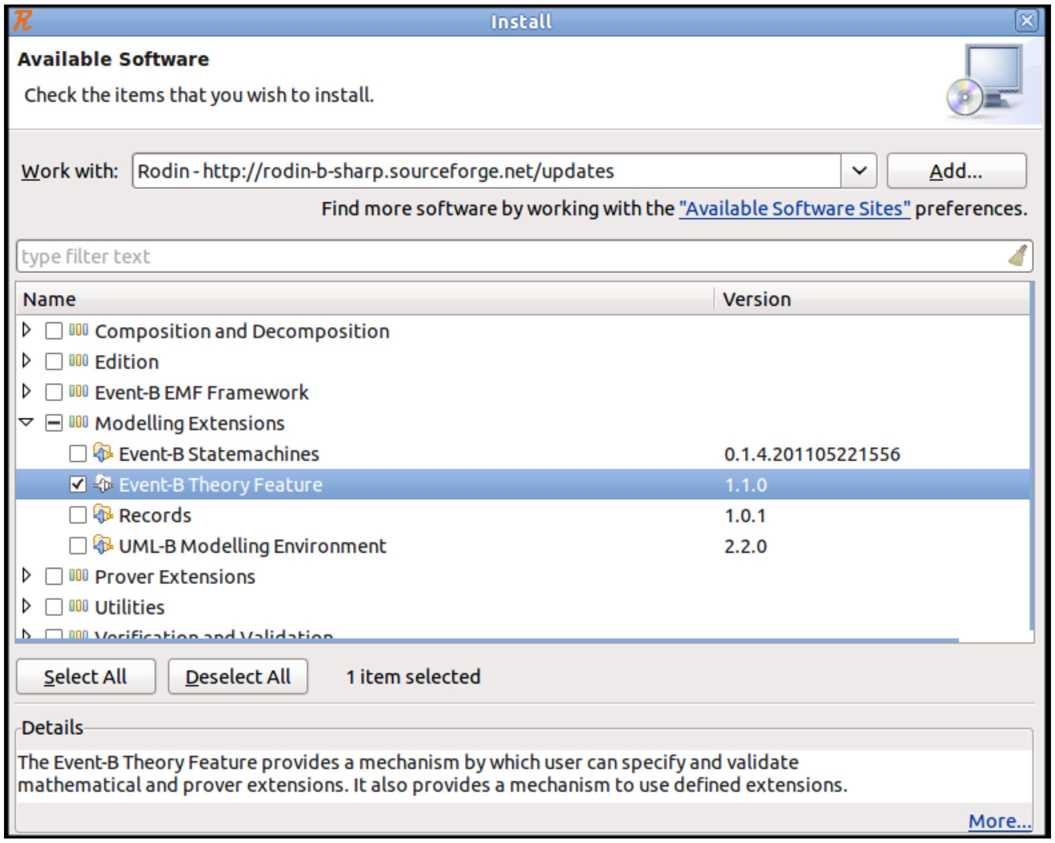
ℹ In the Event-B mathematical language, predicates (known as formulae in most literature) and expressions (known as terms) are two separate syntactic categories. Expressions have a type. Predicate do not.

3 Quick Start

In this section, we step through a simple tutorial on how to define and use a simple theory. Click on the links above to navigate through this tutorial.

3.1 Install Theory Plug-in

Considering figure 3.1, the installation or update for the Theory plug-in is available under the main Rodin Update site (`http://rodin-b-sharp.sourceforge.net/updates`) under the category "Modelling Extensions". Like always, after the installation, restarting Rodin is recommended. For more details, see `http://wiki.event-b.org/index.php/Theory_News_and_Support`.


Install
Available Software
Check the items that you wish to install.
Work with:
Add...
Find more software by working with the "Available Software Sites" preferences.
type filter text
Select All
Deselect All
More...

Rodin - http://rodin-b-sharp.sourceforge.net/updates

Name	Version
Composition and Decomposition	
Edition	
Event-B EMF Framework	
Modelling Extensions	
Event-B Statemachines	0.1.4.201105221556
✓ Event-B Theory Feature	1.1.0
Records	1.0.1
UML-B Modelling Environment	2.2.0
Prover Extensions	
Utilities	
Verification and Validation	

1 item selected

Details

The Event-B Theory Feature provides a mechanism by which user can specify and validate mathematical and prover extensions. It also provides a mechanism to use defined extensions.

Figure 1: Install Theory Plug-in

Once the Theory plug-in is successfully installed, menu entries will be added in certain places, see figure 3.1. In particular, the Event-B Explorer will have an additional button that initiates the wizard to create new theory files.

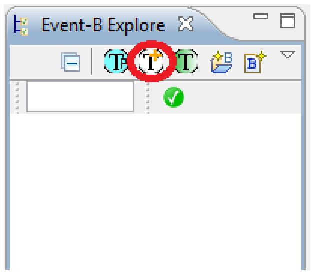
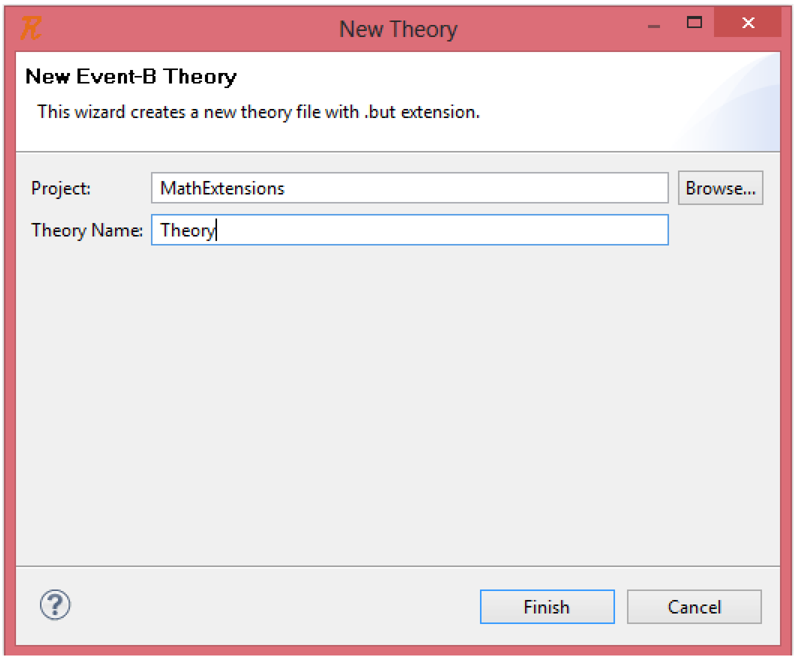


Figure 2: New Theory Button in Event-B Explorer

3.2 Create A New Theory

An additional button (red-circled in figure 3.1) should appear in the Event-B Explorer. By clicking the button, a wizard that enables the creation of a new theory is initiated. Figure 3.2 shows the wizard in action.



New Event-B Theory

This wizard creates a new theory file with .but extension.

Project: MathExtensions

Theory Name: Theory

Figure 3: New Theory File Wizard

In the wizard, specify the parent project of the theory and a theory name. The project can be selected using the button on the right hand side of project name text field (akin to selecting a project when creating a new Event-B component). Click the Finish button to create the theory. If there are no name clashes between the name of the new theory and any existing resources, you should get a theory editor opened up as depicted in figure 3.2.

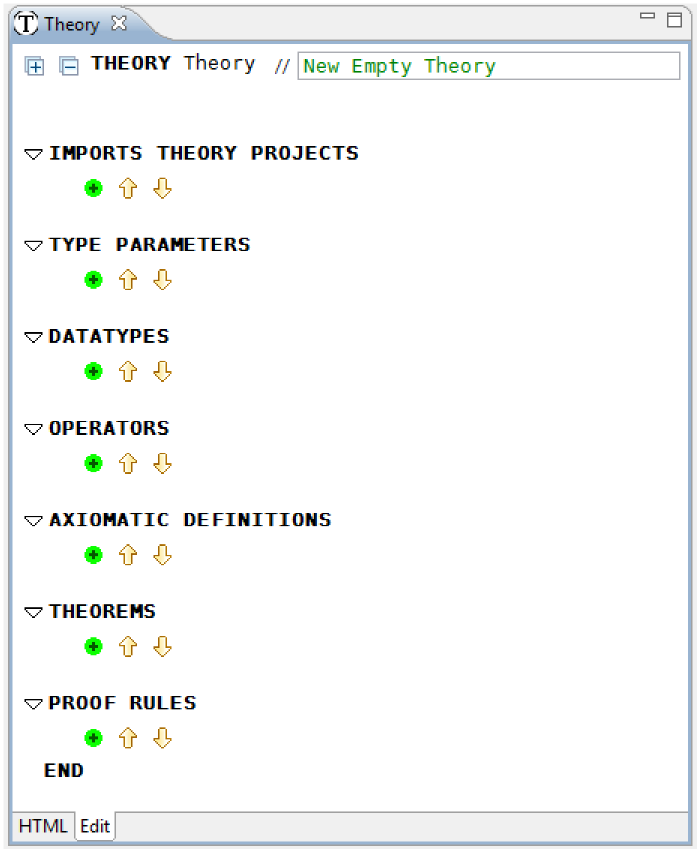


Figure 4: The Theory Editor

The theory editor has two pages: an Edit page and an HTML (i.e., pretty print) page. The edit page corresponds to the structured editor. The HTML page is a pretty print view of the theory.

3.3 Add a Type Parameter

Type parameters in a theory specify the types on which new definitions and rules may be polymorphic. For instance, a theory of sequences can be polymorphic on one type and that is the type of elements it may hold.

Type parameters are similar in nature to carrier sets in contexts. To create a new type parameter, click on button ⊕ under the Type Parameters section of the structured editor, and specify the name of the type parameter (see figure 3.3).

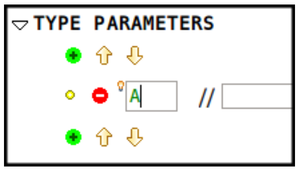


Figure 5: Type Parameters

⚠ Type parameters are expected to be a legal Event-B identifier. For example, $2ident$ is not a legal identifier.

✓ As a convention, type parameters are specified as upper case strings (same as context's carrier sets).

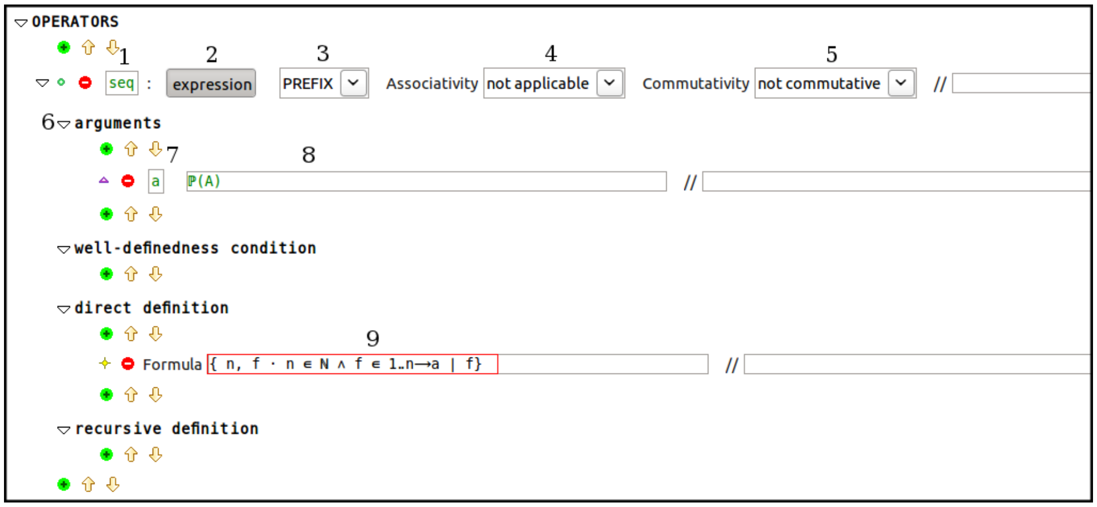
3.4 Create an Operator Definition

Event-B mathematical language has many useful operators. Examples include cardinality operator $card$, the finiteness predicate operator $finite$ and the function application .(.). Other useful operators can be defined using the Theory plug-in. Figure 3.4 shows a definition of a sequence operator.

Note the numbering in figure 3.4. The following explains each part of the definition:

1. **Syntax Symbol**: this specifies the syntax token that will be reserved for the new operator (sequence in our example). It should not clash with any previously defined symbol in the mathematical language available to the theory.

2. **Syntactic Class**: this specifies whether the new operator is an expression operator or a predicate operator. For example, the cardinality operator $card$ is an expression operator of integer type, and the finiteness operator $finite$ is a predicate operator. In our case, the sequence operator is an expression operator. The button can be toggled off if a predicate operator is required instead.



```
OPERATORS
  seq : expression  PREFIX  Associativity not applicable  Commutativity not commutative  //
  arguments
    a  ℙ(A)  //
  well-definedness condition
  direct definition
    Formula { n, f · n ∈ ℕ ∧ f ∈ 1‥n→a | f}  //
  recursive definition
```

Figure 6: Sequence Operator Definition

3. **Notation**: this specifies whether the symbols is a prefix or an infix operator. In the existing mathematical language, $+$ is an infix operator whereas *partition* is a prefix predicate operator. In our example, the sequence operator is specified as prefix.

4. **Associativity**: this specifies whether the operator is associative. Note that this has semantic implications, and as such a proof obligation is generated to check the associativity property.

5. **Commutativity**: this specifies whether the operator is commutative. Note that this has semantic implications, and as such a proof obligation is generated to check the commutativity property.

6. **Operator Arguments**: an operator may have a number of arguments (all of which may be expressions).

7. **Argument Identifier**: this specifies the name of the argument of the operator. It has to be a legal Event-B identifier (similar to carrier sets, constant, variables etc.).

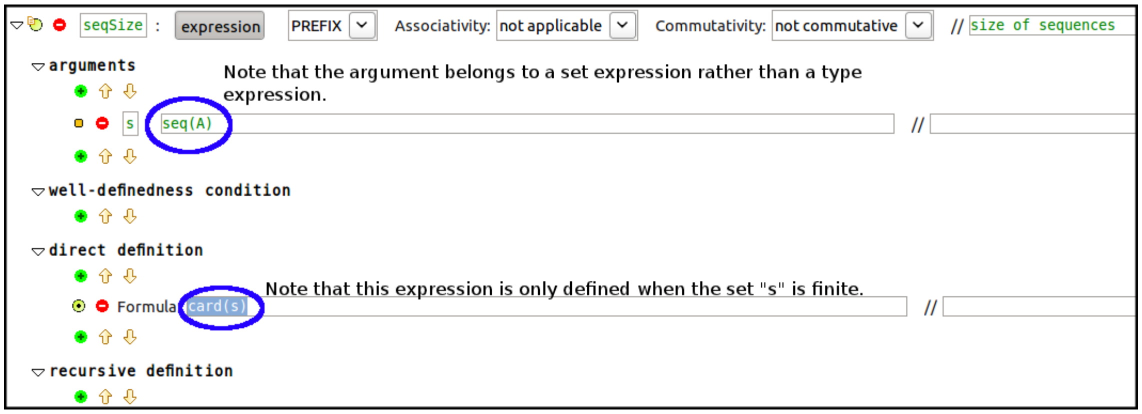
8. **Argument Type**: this specifies the type of the argument. In our case, the sequence operator takes a set of type A. Since A is a type parameter, the sequence is polymorphic.

9. **Direct Definition**: this provides the direct definition of the operator. In our case (see the red-boxed field), it asserts that sequences are total functions from a contiguous integer range starting at 1 to the set a the argument of the operator *seq*.

- Only operators that take two arguments of the same type can be tagged as commutative. Of course, then, one has to prove the mathematical property.
- An operator can be tagged as being associative if it satisfies the three conditions: (1) it is an expression operator, (2) it takes two (or more) arguments of the same type, (3) the type of the operator is the same as that of its arguments. Of course, then, one has to prove the mathematical property.
- Operators that are tagged associative have to be tagged as infix as well.
- The argument type can be a type or a set expression. If the argument type is a set expression, then the type of the argument is inferred. Furthermore, the additional restriction (i.e., that the argument belongs to a set expression) is added as a well-definedness condition for the operator.

- As a convention, names of operators should be lower case.
- As a convention, names of operator arguments should be lower case.

The operator in figure 3.4 defines size for sequences.


seqSize : expression PREFIX Associativity: not applicable Commutativity: not commutative // size of sequences

arguments

Note that the argument belongs to a set expression rather than a type expression.

s seq(A) //

well-definedness condition

direct definition

Note that this expression is only defined when the set "s" is finite.

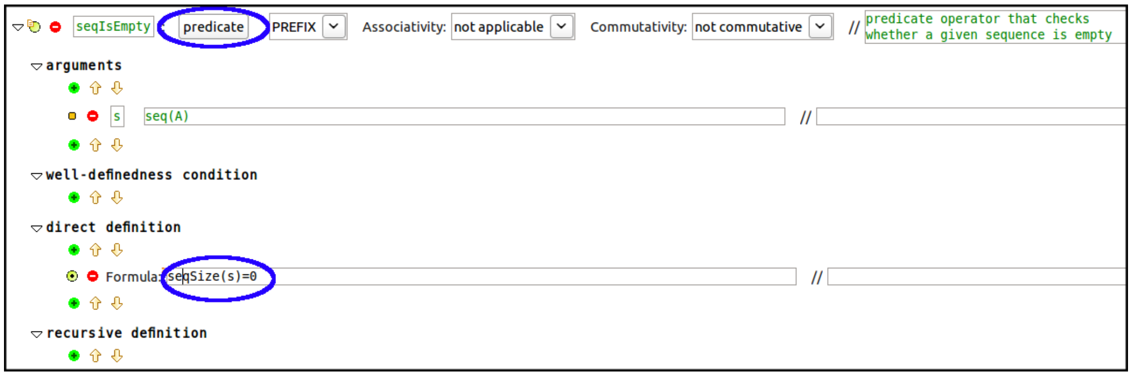
Formula card(s) //

recursive definition

Figure 7: Sequence Size Operator Definition

This definition asserts that the operator $seqSize$ takes one argument of type $\mathbb{Z} \leftrightarrow A$. This definition, also, triggers a proof obligation to prove the strength of the well-definedness condition provided. Namely, one has to prove that $\forall s \cdot s \in seq(A) \Longrightarrow finite(s)$. We leave this as an exercise to the reader.

Figure 3.4 is a definition of a predicate operator:

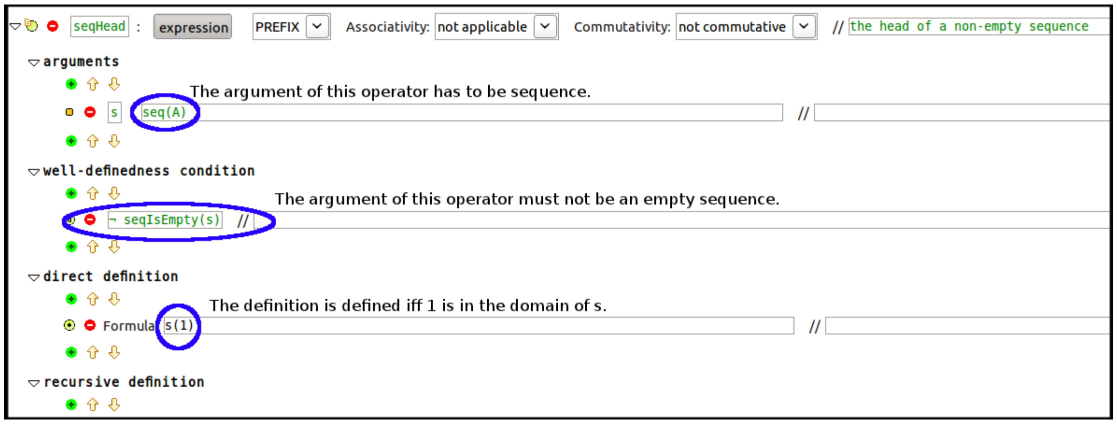


```
seqIsEmpty  predicate  PREFIX  Associativity: not applicable  Commutativity: not commutative  // predicate operator that checks
                                                                                                 whether a given sequence is empty
arguments
    s  seq(A)  //
well-definedness condition
direct definition
    Formula: seqSize(s)=0  //
recursive definition
```

Figure 8: A Predicate Operator Definition

The definition of $seqIsEmpty$ does not trigger any proof obligation for well-definedness strength. This is due to the fact that the corresponding condition is a trivial predicate, namely: $\forall s \cdot s \in seq(A) \implies s \in seq(A)$.

The head operator on sequences can be defined as in figure 3.4.

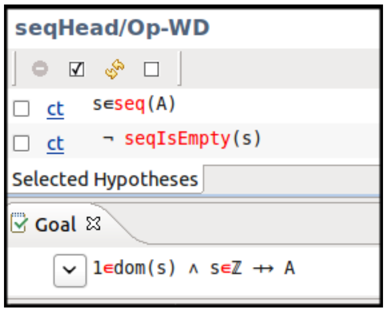


```
seqHead : expression  PREFIX  Associativity: not applicable  Commutativity: not commutative  // the head of a non-empty sequence
arguments
                The argument of this operator has to be sequence.
    s  seq(A)  //
well-definedness condition
                          The argument of this operator must not be an empty sequence.
    ¬ seqIsEmpty(s)  //
direct definition
                The definition is defined iff 1 is in the domain of s.
    Formula s(1)  //
recursive definition
```

Figure 9: Sequence Head Operator Definition

Figure 3.4 shows the well-definedness strength proof obligation corresponding to the previous definition of $seqHead$.

As a summary, have a look at figure 3.4 which is taken from the HTML view of our theory.


seqHead/Op-WD

```
☐ ct   s∈seq(A)
☐ ct     ¬ seqIsEmpty(s)
```

Selected Hypotheses

Goal

```
1∈dom(s) ∧ s∈ℤ ⇸ A
```

Figure 10: Sequence Head WD Strength PO

The proof obligations associated with an operator definition are the following:

1. **./Op-WD** operator well-definedness strength if a well-definedness condition is explicitly specified.
2. **./Op-COMMUT** the commutativity proof obligation, generated if the operator is tagged as commutative.
3. **./Op-ASSOC** the associativity proof obligation, generated if the operator is tagged as associative.

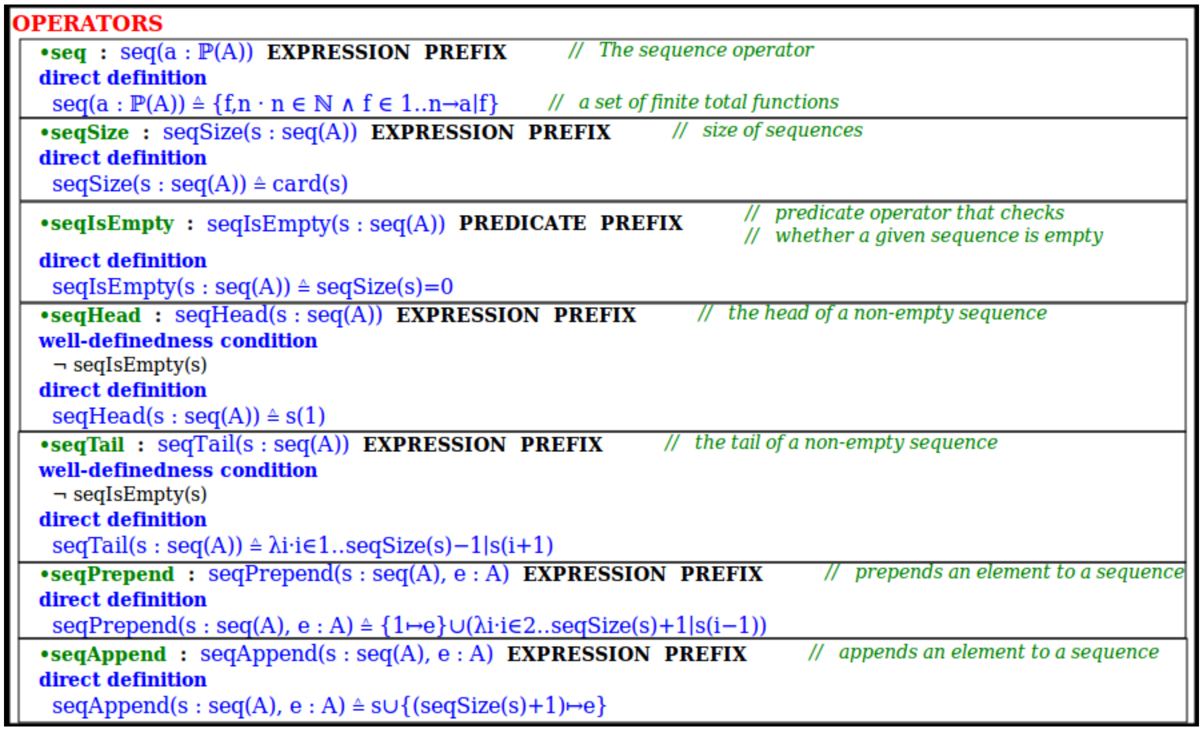
3.5 Specify a Polymorphic Theorem

A polymorphic theorem is no different, in principle, from theorems defined in contexts and machines. The Theory plug-in, however, provides facilities to instantiate and use these theorems in proofs. See the example in figure 3.5.

The previous theorem articulates the fact the sequences as specified in our example are finite. As with theorems in contexts and machines, you have to prove validity and well-definedness of the theorem. The proof obligations associated with a theorem are the following:

1. **./S-THM** the validity proof obligation.
2. **./WD-THM** the well-definedness proof obligation.

Figure 3.5 shows other theorems that can be defined in relation to our theory of sequences so far:



```
OPERATORS
 •seq : seq(a : ℙ(A)) EXPRESSION PREFIX        // The sequence operator
 direct definition
  seq(a : ℙ(A)) ≙ {f,n · n ∈ ℕ ∧ f ∈ 1..n→a|f}   // a set of finite total functions
 •seqSize : seqSize(s : seq(A)) EXPRESSION PREFIX        // size of sequences
 direct definition
  seqSize(s : seq(A)) ≙ card(s)
 •seqIsEmpty : seqIsEmpty(s : seq(A)) PREDICATE PREFIX     // predicate operator that checks
                                                           // whether a given sequence is empty
 direct definition
  seqIsEmpty(s : seq(A)) ≙ seqSize(s)=0
 •seqHead : seqHead(s : seq(A)) EXPRESSION PREFIX       // the head of a non-empty sequence
 well-definedness condition
  ¬ seqIsEmpty(s)
 direct definition
  seqHead(s : seq(A)) ≙ s(1)
 •seqTail : seqTail(s : seq(A)) EXPRESSION PREFIX       // the tail of a non-empty sequence
 well-definedness condition
  ¬ seqIsEmpty(s)
 direct definition
  seqTail(s : seq(A)) ≙ λi·i∈1..seqSize(s)−1|s(i+1)
 •seqPrepend : seqPrepend(s : seq(A), e : A) EXPRESSION PREFIX     // prepends an element to a sequence
 direct definition
  seqPrepend(s : seq(A), e : A) ≙ {1↦e}∪(λi·i∈2..seqSize(s)+1|s(i−1))
 •seqAppend : seqAppend(s : seq(A), e : A) EXPRESSION PREFIX      // appends an element to a sequence
 direct definition
  seqAppend(s : seq(A), e : A) ≙ s∪{(seqSize(s)+1)↦e}
```

Figure 11: Various Sequence Operators

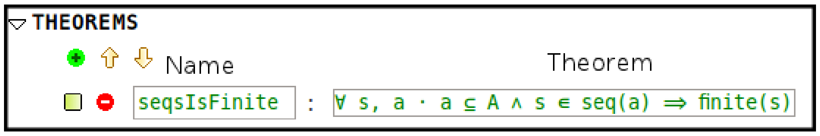
✓ A theorem can be instantiated (e.g., in the previous example, the type parameter A can be substituted with a type expression that is acceptable in the context of the sequent under consideration). We will later show how this is achieved.

3.6 Specify an Inference Rule

Inference rules are proof rules that can be used to: (1) infer new hypotheses in a proof, or (2) split the goal into sub-goals, or (3) discharge a proof obligation. The general structure of an inference rule is as follows:

$$\begin{array}{ll} Given & \\ & G0,\ ...,\ Gn \\ Infer & \\ & I \end{array}$$

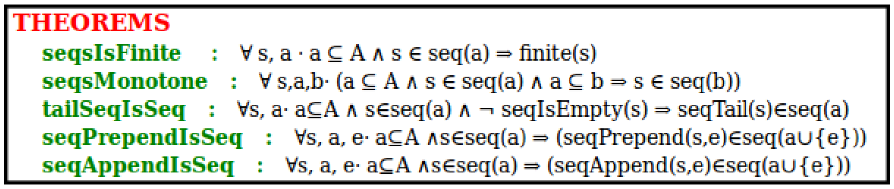
where each of $G0, ..., Gn$ and I is an Event-B predicate. The above inference rule can be read in the following two ways : "given conditions $G0$, ..., Gn one


THEOREMS

Name : Theorem

seqsIsFinite : ∀ s, a · a ⊆ A ∧ s ∈ seq(a) ⇒ finite(s)

Figure 12: Sequence Finiteness Theorem


THEOREMS

seqsIsFinite : ∀ s, a · a ⊆ A ∧ s ∈ seq(a) ⇒ finite(s)

seqsMonotone : ∀ s,a,b· (a ⊆ A ∧ s ∈ seq(a) ∧ a ⊆ b ⇒ s ∈ seq(b))

tailSeqIsSeq : ∀s, a· a⊆A ∧ s∈seq(a) ∧ ¬ seqIsEmpty(s) ⇒ seqTail(s)∈seq(a)

seqPrependIsSeq : ∀s, a, e· a⊆A ∧s∈seq(a) ⇒ (seqPrepend(s,e)∈seq(a∪{e}))

seqAppendIsSeq : ∀s, a, e· a⊆A ∧s∈seq(a) ⇒ (seqAppend(s,e)∈seq(a∪{e}))

Figure 13: Sequence Theorems

can infer I", and "one can prove I, if one can prove each of $G0$, ..., Gn".

Inference rules can be defined as part of a block of "Proof Rules". Each proof rules block may contain a number of metavariables, rewrite rules and inference rules, see figure 3.6. To create a rules block, under the heading "PROOF RULES" in the structured editor, press ⊕.

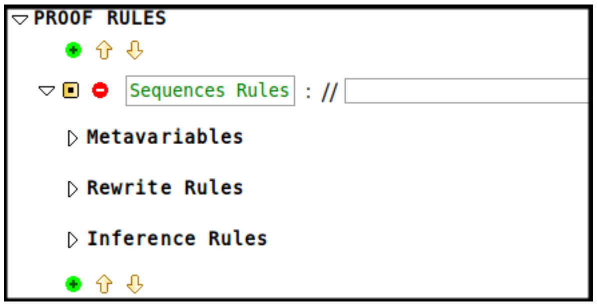


Figure 14: Proof Rules Block

Metavariables define the variables used to specify proof rules. Considering figure 3.6, each metavariable has a name and a type. For our example, we need one metavariable s.

The example in figure 3.6 shows an inference rule concerning finiteness of sequences:

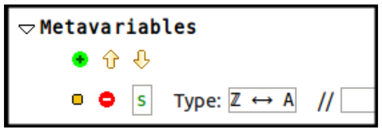


Figure 15: Defining a Metavariable

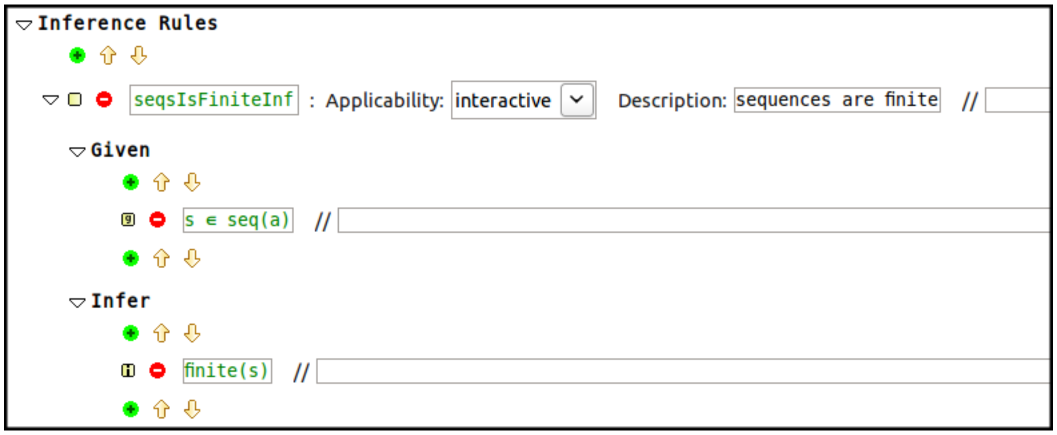
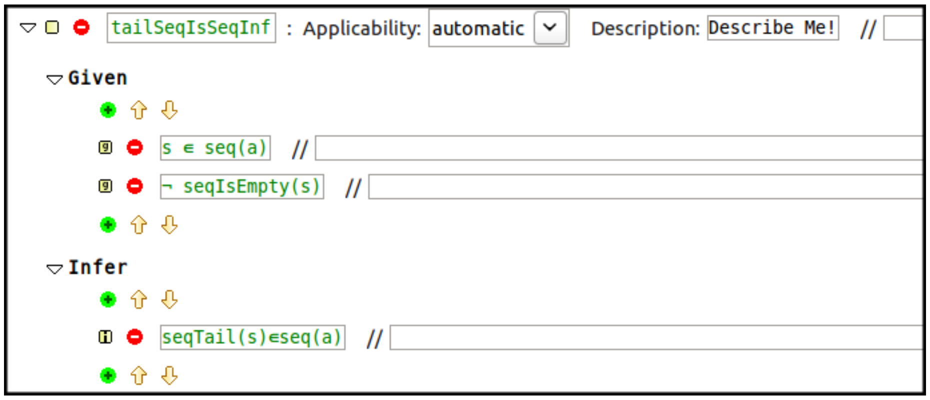


Figure 16: Sequence Finiteness Inference Rule

The applicability of a proof rule indicates whether the rule should be applied automatically or interactively. The description provides a human-readable view of the rule. The description provided will be the tool tip for the rule in the proof UI. The inference rule in figure 3.6 is an automatic rule that states that the tail of a non-empty sequence is a sequence.



```
tailSeqIsSeqInf : Applicability: automatic   Description: Describe Me!  //
Given
    s ∈ seq(a)  //
    ¬ seqIsEmpty(s)  //
Infer
    seqTail(s)∈seq(a)  //
```

Figure 17: Sequence Tail Inference Rule

4 Scoping and Using of Theories

A *theorypath* is a means to introduce the deployed theories in a project scope.
In order to use a math extension, defined in a theory, in a project scope:

1. **Deploy the theory** by clicking the button , figure 1.

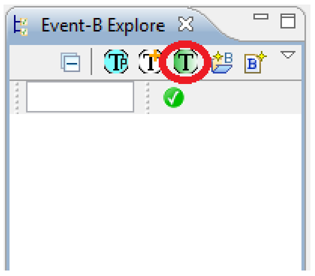


Figure 18: Deploy Theory Button in Event-B Explorer

   Alternatively, a theory can be deployed by a pop-up menu by right-clicking on the thoery.

2. **Import the deployed theory in a *theorypath***, which is defined in the project scope. By clicking the button , a wizard that enables the creation of a new *theorypath* is initiated (See figure 2).

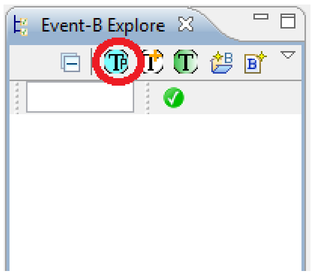


Figure 19: New Theorypath Button in Event-B Explorer

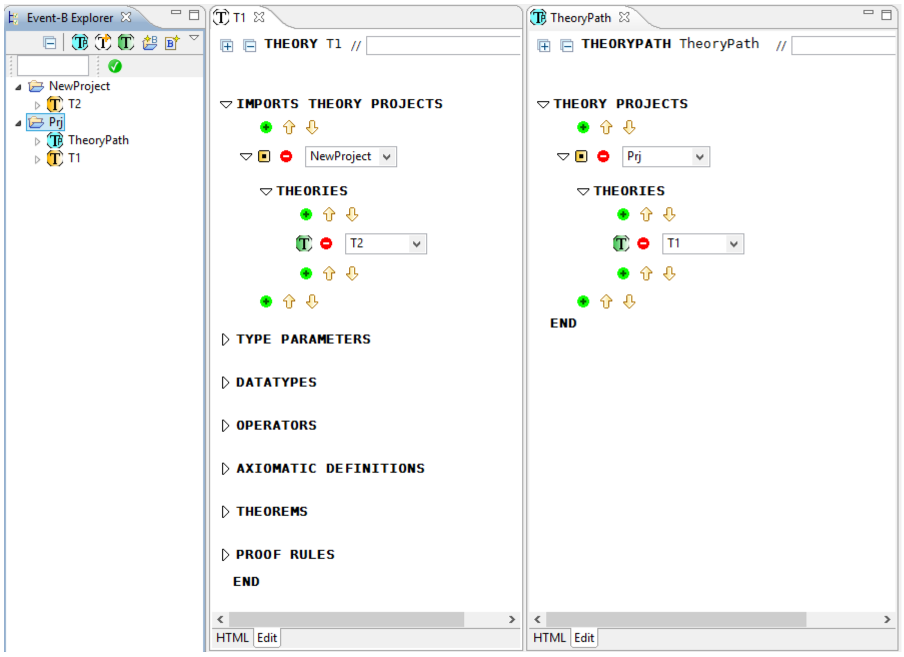
⚠

- A machine/contex accesses (local/global) theories imported directly or indirectly by a *theorypath* within the same project as the machine/contex.
- A theory path can imports deployed (local/global) theories.

- A theory can import deployed (local/global) theories.

- A *local* theory in a project scope is a theory defined inside a project; Whereas a *global* theory is a theory defined in a separated project.

- As illustrated in figure 4, if theory *T1* imports theory *T2*; and theory *T1* is imported in a *theorypath* created in the project *Prj*; Then *T1* is directly and *T2* is indirectly accessible in the *Prj* scope.



```
THEORY T1 //
IMPORTS THEORY PROJECTS
    NewProject
        THEORIES
            T2
TYPE PARAMETERS
DATATYPES
OPERATORS
AXIOMATIC DEFINITIONS
THEOREMS
PROOF RULES
END
```

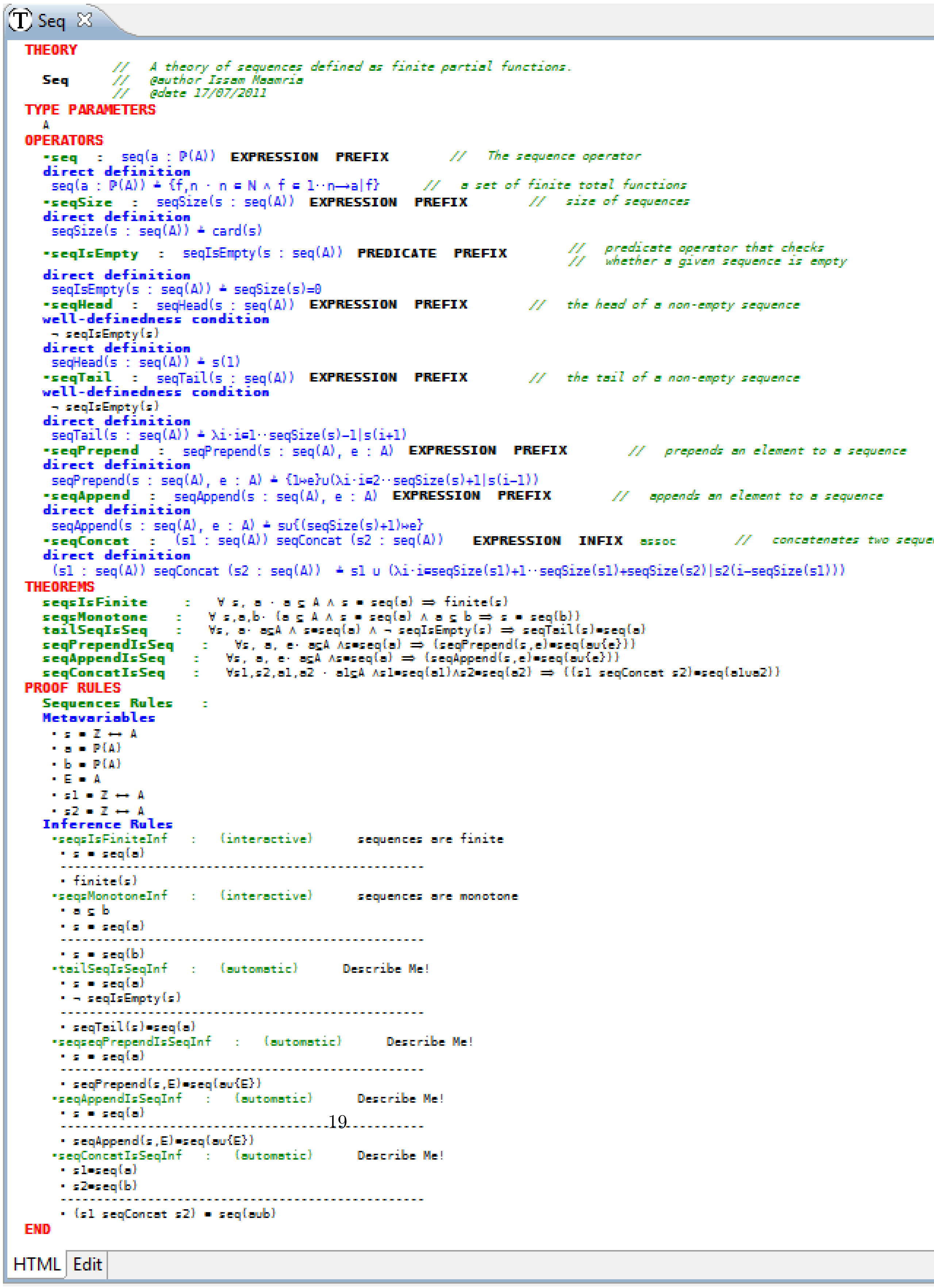
```
THEORYPATH TheoryPath //
THEORY PROJECTS
    Prj
        THEORIES
            T1
END
```

Figure 20: Accessibility of Theories

5 Examples

5.1 Sequence

5.2 List



```
THEORY
            //   A theory of sequences defined as finite partial functions.
  Seq       //   @author Issam Maamria
            //   @date 17/07/2011
TYPE PARAMETERS
  A
OPERATORS
  •seq   :   seq(a : ℙ(A))  EXPRESSION   PREFIX         //   The sequence operator
  direct definition
   seq(a : ℙ(A)) ≙ {f,n · n ∈ ℕ ∧ f ∈ 1‥n→a|f}      //   a set of finite total functions
  •seqSize   :   seqSize(s : seq(A))  EXPRESSION   PREFIX         //   size of sequences
  direct definition
   seqSize(s : seq(A)) ≙ card(s)
  •seqIsEmpty   :   seqIsEmpty(s : seq(A))  PREDICATE   PREFIX         //   predicate operator that checks
                                                                       //   whether a given sequence is empty
  direct definition
   seqIsEmpty(s : seq(A)) ≙ seqSize(s)=0
  •seqHead   :   seqHead(s : seq(A))  EXPRESSION   PREFIX         //   the head of a non-empty sequence
  well-definedness condition
   ¬ seqIsEmpty(s)
  direct definition
   seqHead(s : seq(A)) ≙ s(1)
  •seqTail   :   seqTail(s : seq(A))  EXPRESSION   PREFIX         //   the tail of a non-empty sequence
  well-definedness condition
   ¬ seqIsEmpty(s)
  direct definition
   seqTail(s : seq(A)) ≙ λi·i∈1‥seqSize(s)−1|s(i+1)
  •seqPrepend   :   seqPrepend(s : seq(A), e : A)  EXPRESSION   PREFIX         //   prepends an element to a sequence
  direct definition
   seqPrepend(s : seq(A), e : A) ≙ {1↦e}∪(λi·i∈2‥seqSize(s)+1|s(i−1))
  •seqAppend   :   seqAppend(s : seq(A), e : A)  EXPRESSION   PREFIX         //   appends an element to a sequence
  direct definition
   seqAppend(s : seq(A), e : A) ≙ s∪{(seqSize(s)+1)↦e}
  •seqConcat   :   (s1 : seq(A)) seqConcat (s2 : seq(A))    EXPRESSION   INFIX   assoc         //   concatenates two seque
  direct definition
   (s1 : seq(A)) seqConcat (s2 : seq(A))  ≙ s1 ∪ (λi·i∈seqSize(s1)+1‥seqSize(s1)+seqSize(s2)|s2(i−seqSize(s1)))
THEOREMS
  seqsIsFinite     :    ∀ s, a · a ⊆ A ∧ s ∈ seq(a) ⇒ finite(s)
  seqsMonotone     :    ∀ s,a,b· (a ⊆ A ∧ s ∈ seq(a) ∧ a ⊆ b ⇒ s ∈ seq(b))
  tailSeqIsSeq     :    ∀s, a· a⊆A ∧ s∈seq(a) ∧ ¬ seqIsEmpty(s) ⇒ seqTail(s)∈seq(a)
  seqPrependIsSeq    :    ∀s, a, e· a⊆A ∧s∈seq(a) ⇒ (seqPrepend(s,e)∈seq(a∪{e}))
  seqAppendIsSeq    :    ∀s, a, e· a⊆A ∧s∈seq(a) ⇒ (seqAppend(s,e)∈seq(a∪{e}))
  seqConcatIsSeq    :    ∀s1,s2,a1,a2 · a1⊆A ∧s1∈seq(a1)∧s2∈seq(a2) ⇒ ((s1 seqConcat s2)∈seq(a1∪a2))
PROOF RULES
  Sequences Rules     :
  Metavariables
   • s ∈ ℤ ↔ A
   • a ∈ ℙ(A)
   • b ∈ ℙ(A)
   • E ∈ A
   • s1 ∈ ℤ ↔ A
   • s2 ∈ ℤ ↔ A
  Inference Rules
   •seqsIsFiniteInf    :    (interactive)      sequences are finite
    • s ∈ seq(a)
   -----------------------------------------------------
    • finite(s)
   •seqsMonotoneInf    :    (interactive)      sequences are monotone
    • a ⊆ b
    • s ∈ seq(a)
   -----------------------------------------------------
    • s ∈ seq(b)
   •tailSeqIsSeqInf    :    (automatic)      Describe Me!
    • s ∈ seq(a)
    • ¬ seqIsEmpty(s)
   -----------------------------------------------------
    • seqTail(s)∈seq(a)
   •seqseqPrependIsSeqInf    :    (automatic)      Describe Me!
    • s ∈ seq(a)
   -----------------------------------------------------
    • seqPrepend(s,E)∈seq(a∪{E})
   •seqAppendIsSeqInf    :    (automatic)      Describe Me!
    • s ∈ seq(a)
   -----------------------------------------------------
    • seqAppend(s,E)∈seq(a∪{E})
   •seqConcatIsSeqInf    :    (automatic)      Describe Me!
    • s1∈seq(a)
    • s2∈seq(b)
   -----------------------------------------------------
    • (s1 seqConcat s2) ∈ seq(a∪b)
END
```