EB[ASTD]: Meta-modelling framework for ASTD

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Outline

Motivation

ASTD is a graphical notation that combines

- Statechart : hierarchy, AND-states, OR-states..
- Process algebra : concurrency, compositionality

which allows

- **modularity**
- **expressiveness**
- adding variables
- an operational semantics

How to reason on ASTD models ?

Outline

ASTD – Algebraic State Transition Diagram

ASTD is :

- a graphical *state-based* formalism
- built *incrementaly* by applying **compositional** operator

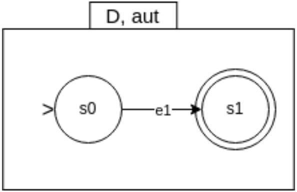

Figure – D = Automaton()

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- built *incrementaly* by applying **compositional** operator
- equipped with *sequent-based* transition rules [5, 6]
- applied on intrusion detection and autonomous vehicle [14, 4, 11]

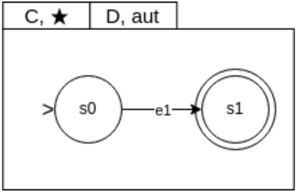

Figure – C = Closure(D)

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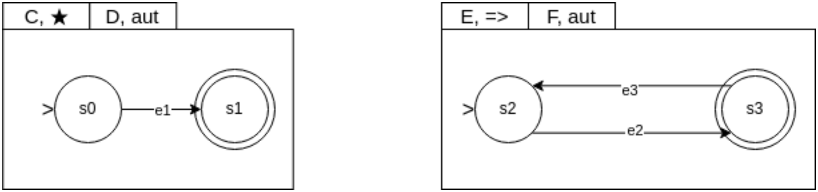

Figure – Two ASTDs C and E

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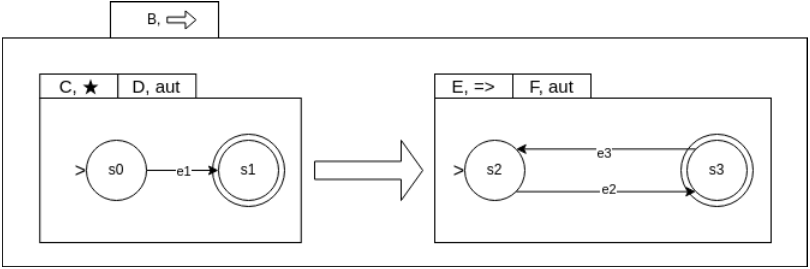

Figure – B = Sequence(C,E)

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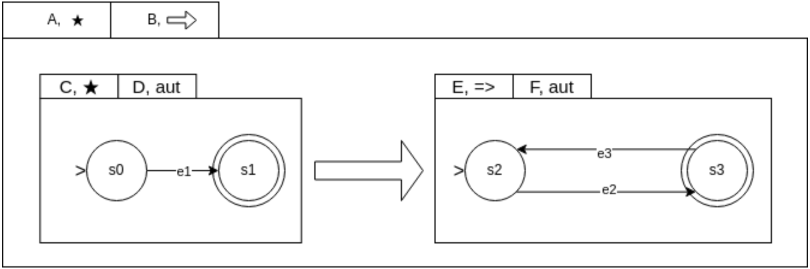

Figure – Final ASTD

Exploit an ASTD specification

Interesting **research directions** based on ASTD features

- graphical formalism $\Longrightarrow$ Visualization
- operational semantics $\Longrightarrow$ Animation
- state variables $\Longrightarrow$ Invariants
- state-based $\Longrightarrow$ Advanced properties (for eg. Liveness)

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All above need a developing environment for ASTD
but developing from scratch is cumbersome...
$\Longrightarrow$ Embedding ASTD in a unified modelling framework

Outline

Existing approaches for ASTDs

cASTD is a *compiler* that generates C++ code based on a given specification. [15, 3]

- + It can ***animate*** an ASTD for **validation** purpose
- - *Poor* animation features (no backtracking, traces, . . .)

ASTD2EB is a *model to text* transformation
where compositional operators become a **pattern** in Event-B. [12]

- + Rodin tool support and proof assistant
- - *Loss* of hierarchy structure

pASTD : provides definition of **invariants** and **POs**. It is a java plugin that transfer POs in Event-B [8]

- + easier to prove
- - loss of traceability
- - POs are defined (but no soundness)

Position of our work...

Despite advances in incorporating features, ASTD tools

- have primarily focused on **design** and **testing**
- based on ***ad hoc* model transformation**
- that lost **hierarchical structure** of the original ASTD

$\Longrightarrow$

- The power of ASTD is ***not* exploited** by target language
- Spec error is **difficult to trace back** to the original ASTD

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Only manipulation of ASTD instances
$\Longrightarrow$ ***mechanize*** its semantics to manipulate ASTDs as first order objects for analysis (animation, ..), reasoning (proof,..)

Formalised embedding approaches

We are inspired from **meta-modelling** approaches, such as :

- MetaCoq project to build a meta-programming environment, implemented in CertiCoq [19, 2]
- Modelling HOL models in Isabelle/HOL [9]
- B models embedded in PVS [13]
- Event-B Machine encoded in CSP [10, 18]
- **EB4EB**, a reflexive framework for advanced **reasoning** and **analysis** on Event-B machine in Event-B [17, 16]

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In the same spirit of EB4EB, we *invite* the semantics of ASTD in Event-B as algebraic theory.

Outline

Modelling language – Event-B [1]

Correct-by-construction, state-based formal method, refinement. Two components : Contexts and Machines

CONTEXT Ctx **SETS** s **CONSTANTS** c **AXIOMS** A **THEOREMS** T_{ctx} **END**	**MACHINE** M **SEES** Ctx **VARIABLES** x **INVARIANTS** $I(x)$ **THEOREMS** $T_{mch}(x)$ **VARIANT** $V(x)$ **EVENTS** INITIALISATION $\widehat{=}$ ... $evt \ \widehat{=}$ **ANY** α **WHERE** $G(x, \alpha)$ **THEN** $x \mid BAP(\alpha, x, x')$ **END** **END**

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Theorems Context (*THM_ctx*)	$A \Rightarrow T_{ctx}$
Theorems machine (*THM_mch*)	$A \wedge I(x) \Rightarrow T_{mch}(x)$
Initialisation (INIT)	$A \wedge AP(\alpha, x') \Rightarrow I(x')$
Invariant preservation (INV)	$A \wedge I(x) \wedge G(x, \alpha) \wedge BAP(x, \alpha, x') \Rightarrow I(x')$
Event feasibility (FIS)	$A \wedge I(x) \wedge G(x, \alpha) \Rightarrow \exists x' \cdot BAP(x, \alpha, x')$
Variant progress (VAR)	$A \wedge I(x) \wedge G(x, \alpha) \wedge BAP(x, \alpha, x') \Rightarrow V(x') < V(x)$

Event-B extension [7]

Formalisation of new data-types for Event-B $\Longrightarrow$ Theories

```
THEORY Th
IMPORT Th1, ...




END
```

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```
THEORY Th
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TYPE PARAMETERS E, F, ...

DATATYPES
  Type2(E, ...)
  constructors
    cstr1(p1 : T1, ...)

OPERATORS
  Op1<nature> (p1 : T1, ...)

    direct definition D1

END
```

- Algebraic definition for data-types $\Longrightarrow$ Constructive definitions

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- Algebraic definition for data-types $\Longrightarrow$ Constructive definitions and/or axiomatic definitions.

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- Well-Definedness (WD) conditions (partial definitions).

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THEOREMS T1, ...
PROOF RULES. . .

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- Algebraic definition for data-types $\Longrightarrow$ Constructive definitions and/or axiomatic definitions.
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- Relevant inference/rewrite rules.

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- Theory Plugin for Rodin environment.

Outline

ASTDStruct - Import the syntax

Constructive datatype.

- Statename, event, variable
- ASTD operators integrated
- Focus on Sequence

$P(x)$ denoted as $\{x|P(x)\}$
axm of comprehension

```
THEORY ASTDStruct
  TYPE PARAMETERS St, Ev, Var
  DATA TYPES
    ASTD(St, Ev, Var)
    constructors
    Elementary(...)
    Automaton(...)
    Sequence(
      SeqFirst : ASTD(St, Ev, Var),
      SeqSnd : ASTD(St, Ev, Var),
      SeqAttr : ℙ(Var),
      SeqInitAttr : ℙ(Var × ℤ) ↔ ℙ(Var × ℤ),
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      )
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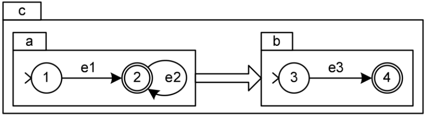

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ASTDStruct - Well definedness conditions

WD conditions are introduced in each ASTD operator

- Constraints on actions, scope, type..

- Group in a single operator

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OPERATORS
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ASTD_WellTyped ***predicate*** $(a : ASTD(St, Ev, Var))$

Scope_WellCons ***predicate*** $(a : ASTD(St, Ev, Var),$
$accVar : \mathbb{P}(Var))$

InitActionS_WellCons ***predicate*** $(a : ASTD(St, Ev, Var))$

InvariantS_WellCons ***predicate*** $(a : ASTD(St, Ev, Var),$
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ASTD_WellCons ***predicate***
$(a : ASTD(St, Ev, Var), accVar : \mathbb{P}(Var))$
direct definition
$Automaton\_WellCons(a)$
$\wedge ASTD\_WellTyped(a)$
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ASTDBehavior - Transition rules

Inductive sequent-based operational semantics rules. (For Eg. aut_2)

NextState ***expression*** (
    astd : $ASTD(St, Ev, Var)$,
    current : $ASTDState(St, Var)$,
    $E_e : Var \nrightarrow \mathbb{Z}$)
well–defnedness condition
    $ASTD\_WellCons(astd, dom(E_e))$
recursive definition
case astd :
  Elementary(inv) => ...
  Automaton$(i, f, ..., mapping)$ =>
    $aut_1 \cup \mathbf{aut_2} \cup aut_3$
  Sequence$(fst, snd, attr, initAttr, inv)$
      => ....
  Closure(..) => ...
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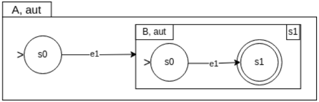

$$\frac{s \xrightarrow{\sigma, E_g, E'_g}_{a.\nu(n)} s' \qquad \begin{array}{l} E_g = E_e \lhd\!\!- E \\ E'_e = E_e \lhd\!\!- attr \lhd E'_g \\ E' = attr \lhd E'_g \end{array}}{(aut_\circ, n, E, s) \xrightarrow{\sigma, E_e, E'_e}_{a} (aut_\circ, n, E', s')}\ aut_2$$

$\{aut_\circ(n, E', s') \mapsto E'_e \mid n, E, s, s', E'_e, E_g, E'_g, E' \cdot$
2. $\wedge n = StateName(current)$
3. $\wedge s = AutSubState(current)$
4. $\wedge E = AutCache(current)$
5. $\wedge s' \mapsto E'_g \in NextState(mapping(n), \sigma, s, E_g)$
6. $\wedge E_g = E_e \lhd\!\!- E$
7. $\wedge E'_e = E_e \lhd\!\!- attr \lhd E'_g$
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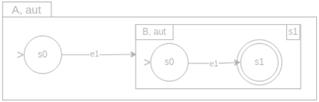

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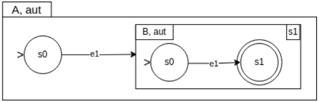

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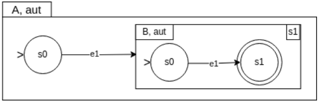
NextState ***expression*** (
  $astd$: $ASTD(St, Ev, Var)$,
  $current$: $ASTDState(St, Var)$,
  E_e : $Var \nrightarrow \mathbb{Z}$)
well-defindness condition
  $ASTD\_WellCons(astd, dom(E_e))$
recursive definition
case astd :
  Elementary(inv) => ...
  Automaton($i, f, ..., mapping$) =>
    $aut_1 \cup$ **aut₂** $\cup\, aut_3$
  Sequence($fst, snd, attr, initAttr, inv$)
    => ....
  Closure(..) => ...
  Guard(...) => ...



$$aut_2\ \frac{s \xrightarrow{\sigma, E_g, E'_g}_{a.\nu(n)} s' \qquad E_g = E_e \mathbin{\lhd\!\!\!+} E \qquad E'_e = E_e \mathbin{\lhd\!\!\!+} attr \mathbin{\lhd\!\!\!-} E'_g \qquad E' = attr \lhd E'_g}{(aut_\circ, n, E, s) \xrightarrow{\sigma, E_e, E'_e}_{a} (aut_\circ, n, E', s')}$$

$\{aut_\circ(n, E', s') \mapsto E'_e \mid n, E, s, s', E'_e, E_g, E'_g, E' \cdot$
2. $\wedge n = StateName(current)$
3. $\wedge s = AutSubState(current)$
4. $\wedge E = AutCache(current)$
5. $\wedge s' \mapsto E'_g \in NextState(mapping(n), \sigma, s, E_g)$
6. $\wedge E_g = E_e \mathbin{\lhd\!\!\!+} E$
7. $\wedge E'_e = E_e \mathbin{\lhd\!\!\!+} attr \mathbin{\lhd\!\!\!-} E'_g$
8. $\wedge E' = attr \lhd E'_g$
$\}$

ASTDBehavior - Transition rules

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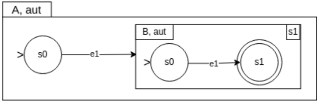

$$aut_2 \; \frac{s \xrightarrow{\sigma, E_g, E'_g}_{a.\nu(n)} s' \qquad E_g = E_e \mathbin{<\!\!+} E \quad E'_e = E_e \mathbin{<\!\!+} attr \lhd E'_g \quad E' = attr \lhd E'_g}{(aut_\circ, n, E, s) \xrightarrow{\sigma, E_e, E'_e}_a (aut_\circ, n, E', s')}$$

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      ASTD_WellCons(astd, dom(E_e))
recursive definition
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   Elementary(inv) => ...
   Automaton(i, f, ..., mapping) =>
      aut_1 ∪ aut_2 ∪ aut_3
   Sequence(fst, snd, attr, initAttr, inv)
           => ....
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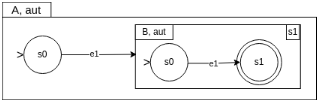

$$aut_2 \quad \frac{s \xrightarrow{\sigma, E_g, E'_g}_{a.\nu(n)} s' \qquad \begin{array}{l} E_g = E_e \Lsh E \\ E'_e = E_e \Lsh attr \lhd E'_g \\ E' = attr \lhd E'_g \end{array}}{(aut_\circ, n, E, s) \xrightarrow{\sigma, E_e, E'_e}_{a} (aut_\circ, n, E', s')}$$

$\{aut_\circ(n, E', s') \mapsto E'_e \mid n, E, s, s', E'_e, E_g, E'_g, E' \cdot$

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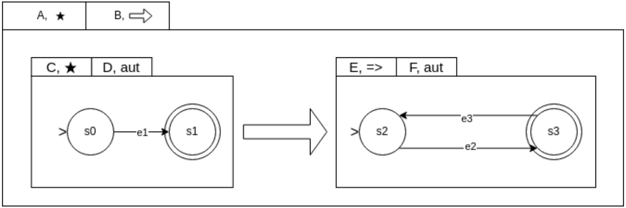
$\}$

Outline

Modelling specific ASTD as an instance

Back to the example



State variable, actions, invariant are defined :

ASTD	Attributes	Initialisation	Invariant
A	x_A	$x_A := 0$	$x_A \geq 0$
B	x_B	$x_B := x_A + 1$	$x_B > 0$
...			
S3			$x_F > 0 \wedge x_A > 2 * x_E$

Deep Instantiation

CONTEXT IllustrativeExample
CONSTANTS
e1, e2, e3,
s0, s1, s2, s3,
astd, B, C, D, E, F,
xA, xB, xC, xD, xE, xF
AXIOMS
axm1: $partition(Ev, \{e1\}, \{e2\}, \{e3\})$
axm2: $partition(St, \{s0\}, \{s1\}, \{s2\}, \{s3\})$
axm3: $partition(Var, \{xA\}, \ldots, \{xF\})$
axm4: $root \in ASTD(St, Ev, Var)$
axm12: $F = Automaton(s2, \{s3\}, \ldots)$
axm13: $E = Guard(\{\ldots\}, F, \{xE\}, \ldots\})$
axm14: $D = Automaton(\ldots)$
axm15: $C = Closure(D, \ldots)$
axm16: $B = Sequence(C, E, \ldots)$
axm17: $root = Closure(B, \ldots)$
THEOREMS
thm_of_WD: $ASTD\_WellCons(root, \emptyset)$
END

Deep Instantiation

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e1 , e2 , e3 ,
s0 , s1 , s2 , s3 ,
astd , B, C, D, E, F,
xA , xB , xC , xD , xE , xF
AXIOMS
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- Applying datatype constructors as compositional operator

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axm12: F = Automaton(s2, {s3}, ...)
axm13: E = Guard({...}, F, {xE}, ...})
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axm15: C = Closure(D, ...)
axm16: B = Sequence(C, E, ...)
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THEOREMS
thm_of_WD: ASTD_WellCons(root, ∅)
END
```

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Deep Instantiation

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- Applying datatype constructors as compositional operator
- ASTD consistency is ensured by discharging the *THM_ctx* PO generated for the *thm_of_WD* theorem.

$$axm1 \wedge .. \wedge axm17 \implies thm\_of\_WD$$

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ASTD is instantiated as First class citizen

Shallow instantiation

A generic machine that interpret ASTD.

1. The *current* state is initialized by Init
2. The *progress* event

```
MACHINE ShallowGenAnim
SEES ASTD_Ctx
VARIABLES current
INVARIANTS
  inv1 : current ∈ ASTDState(St, Var)
EVENTS
  INITIALISATION
  THEN
    act1 : current := Init(root, ∅)
  END
  progress
  ANY evnt , nxt
  WHERE
    grd1 : evnt ∈ Ev
    grd2 : nxt ∈ NextState(root, evnt, current, ∅)
    grd3 : NextState(root, evnt, current, ∅) ≠ ∅
  THEN
    act1 : current := prj1(nxt)
  END
END
```

Listing 1 – Shallow Instantiation

Shallow Instantiation : Animation

Soundness principle

A proof obligation is a maths formula to be proven associated to the correctness of a given property.

How do I know the proof obligation actually entails the property ?

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Encoding the POs in form of properties on traces.

$ASTD \vdash PO \implies Spec\_\mathbf{[}PO\mathbf{]}\_On\_Traces(ASTD)$

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Invariants case

$ASTD \vdash PO_{INV} \implies Spec\_PO_{INV}\_On\_Traces(ASTD)$

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Invariants case

$ASTD \vdash PO_{INV} \implies Spec\_PO_{INV}\_On\_Traces(ASTD)$

$$\boxed{ASTD \vdash PO_{INV} \implies \underbrace{\forall tr \in Traces(ASTD) : \forall i \geq 0 : INV(tr(i)))}_{Spec\_PO_{INV}\_On\_Traces(ASTD)}}$$

Outline

ASTDPO - Proof Obligations from pASTD

$$ASTD \vdash \boxed{\textbf{PO}} \implies \forall tr \in Traces(ASTD) : \forall i \geq 0 : INV(tr(i)))$$

Two POs are defined :

- PO_{init} : The invariants hold for the initialization
- PO_{trans} : Transitions preserves invariants

Automatic PO generation

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IMPORT THEORY ASTDBehaviour
TYPE PARAMETERS St, Ev, Var
OPERATORS
   POinit predicate (a : ASTD(St, Ev, Var), ...)
     well-definedness condition ....
     recursive definition ...

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     well-definedness condition ....
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```

```
CONTEXT   ASTD_Ctx_PO
EXTENDS   ASTD_Ctx
THEOREMS
   Generation_of_POi : POinit(root, ...)
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```

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CONTEXT   ASTD_Ctx_PO
EXTENDS   ASTD_Ctx
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CONTEXT ASTD_Ctx_PO
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Outline

ASTDTraces - Trace based semantics

$$ASTD \vdash \textbf{PO} \implies \forall tr \in \boxed{\textbf{Traces(ASTD)}} : \forall i \geq 0 : INV(tr(i))$$

A sequence of $s_0 \mapsto \cdots \mapsto s_n$ states

1. s_0 is the initial state, i.e. $s_0 = Init(A, \emptyset)$
2. consecutive states link by triggered event $\forall i, \exists e \in Ev, s_i \xrightarrow{e} s_{i+1}$
3. trace is infinite or finite and deadlock, i.e. total/partial function

```
THEORY ASTDTraces
IMPORT THEORY ASTDPO
TYPE PARAMETERS St , Ev , Var
OPERATORS
IsANextState predicate
        (a : ASTD(St, Ev, Var), s, sp : ASTDState(St, Var))
well-definedness condition ASTD_WellCons(a, ∅)
direct definition ...

IsATrace predicate (a : ASTD(St, Ev, Var) ,
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1−     tr(0) = Init(a, ∅)
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        IsANextState(a, tr(i), tr(j)))
3− ∧ (tr ∈ ℕ → ASTDState(St, Var) ∨ // infinite trace
        (∃n · n ∈ ℕ ∧ tr ∈ 0..n → ASTDState(St, Var)
           ∧ (¬CanProgress(a, tr(n))))) // finite trace
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OPERATORS
IsANextState predicate
        (a : ASTD(St, Ev, Var), s, sp : ASTDState(St, Var))
well−definedness condition ASTD_WellCons(a, ∅)
direct definition ...

IsATrace predicate (a : ASTD(St, Ev, Var) ,
        tr : ℕ ⇸ ASTDState(St, Var))
well−definedness condition ASTD_WellCons(a, ∅)
direct definition
1−    tr(0) = Init(a, ∅)
2− ∧ (∀i, j · i ∈ dom(tr) ∧ j ∈ dom(tr) ∧ j = i + 1 ⇒
        IsANextState(a, tr(i), tr(j)))
3− ∧ (tr ∈ ℕ → ASTDState(St, Var) ∨ // infinite trace
        (∃n · n ∈ ℕ ∧ tr ∈ 0..n → ASTDState(St, Var)
            ∧ (¬CanProgress(a, tr(n))))) // finite trace
```

Specification on astd state

Definition of "holding invariants" for an ASTD state.

$$ASTD \vdash \mathbf{PO} \implies \forall tr \in \mathbf{Traces(ASTD)} : \forall i \geq 0 : \boxed{\mathbf{INV(tr(i))}}$$

```
THEORY ASTDCorrectness
IMPORT THEORY ASTDTraces
TYPE PARAMETERS St, Ev, Var
OPERATORS
  InvSpecOnState predicate (a : ASTD(St, Ev, Var),
      s : ASTDState(St, Var))
    well-definedness condition
    recursive definition
  Elementary(inv) => ...
  Automaton(i, f, ..., mapping) => ...
  Sequence(fst, snd, attr, initAttr, inv) => ....
  Closure(...) => ...
  Guard(...) => ...
```

Listing 2 – Definition of satisfying invariants

Ultimate proof of soundness

$$ASTD \vdash \textbf{PO} \boxed{\Longrightarrow} \forall tr \in \textbf{Traces(ASTD)} : \forall i \geq 0 : \textbf{INV(tr(i))}$$

thm_of_PO_Correctness :
$\forall astd, tr \cdot$

$astd \in ASTD(St, Ev, Var)$ // for all astd
$\wedge ASTD\_WellCons(astd, \emptyset)$ // well cons

$\wedge tr \in \mathbb{N} \nrightarrow ASTDState(St, Var)$ // and trace
$\wedge IsATrace(astd, tr)$ // of the astd

$\wedge POinit(astd, Var \nrightarrow \mathbb{Z}, Var \nrightarrow \mathbb{Z}, (Var \nrightarrow \mathbb{Z}) \lhd id)$
$\wedge POtrans(astd, Var \nrightarrow \mathbb{Z}, \emptyset)$ // when POs hold

$\Rightarrow$
$(\forall i \cdot i \in dom(tr) \Rightarrow InvSpecOnState(astd, tr(i)))$
// every state in the trace satisfies the invariant

Ultimate proof of soundness

$$ASTD \vdash \mathbf{PO} \boxed{\Longrightarrow} \forall tr \in \mathbf{Traces(ASTD)} : \forall i \geq 0 : \mathbf{INV(tr(i))}$$

thm_of_PO_Correctness :
$\forall astd, tr \cdot$

$astd \in ASTD(St, Ev, Var)$ // for all astd
$\wedge ASTD\_WellCons(astd, \emptyset)$ // well cons

$\wedge tr \in \mathbb{N} \rightarrowtail ASTDState(St, Var)$ // and trace
$\wedge IsATrace(astd, tr)$ // of the astd

$\wedge POinit(astd, Var \rightarrowtail \mathbb{Z}, Var \rightarrowtail \mathbb{Z}, (Var \rightarrowtail \mathbb{Z}) \lhd id)$
$\wedge POtrans(astd, Var \rightarrowtail \mathbb{Z}, \emptyset)$ // when POs hold

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$(\forall i \cdot i \in dom(tr) \Rightarrow InvSpecOnState(astd, tr(i)))$
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$(\forall i \cdot i \in dom(tr) \Rightarrow InvSpecOnState(astd, tr(i)))$
// every state in the trace satisfies the invariant

Ultimate proof of soundness

$$ASTD \vdash \mathbf{PO} \boxed{\color{red}{\Longrightarrow}} \forall tr \in \mathbf{Traces(ASTD)} : \forall i \geq 0 : \mathbf{INV(tr(i))}$$

thm_of_PO_Correctness :
$\forall astd, tr \cdot$

$astd \in ASTD(St, Ev, Var)$ // for all astd
$\wedge ASTD\_WellCons(astd, \emptyset)$ // well cons

$\wedge tr \in \mathbb{N} \nrightarrow ASTDState(St, Var)$ // and trace
$\wedge IsATrace(astd, tr)$ // of the astd

$\wedge POinit(astd, Var \nrightarrow \mathbb{Z}, Var \nrightarrow \mathbb{Z}, (Var \nrightarrow \mathbb{Z}) \lhd id)$
$\wedge POtrans(astd, Var \nrightarrow \mathbb{Z}, \emptyset)$ // when POs hold

$\Rightarrow$
$(\forall i \cdot i \in dom(tr) \Rightarrow InvSpecOnState(astd, tr(i)))$
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Ultimate proof of soundness

$$ASTD \vdash \textbf{PO} \boxed{\Longrightarrow} \forall tr \in \textbf{Traces(ASTD)} : \forall i \geq 0 : \textbf{INV(tr(i))}$$

thm_of_PO_Correctness :
$\forall astd, tr \cdot$

$astd \in ASTD(St, Ev, Var)$ // for all astd
$\wedge ASTD\_WellCons(astd, \emptyset)$ // well cons

$\wedge tr \in \mathbb{N} \nrightarrow ASTDState(St, Var)$ // and trace
$\wedge IsATrace(astd, tr)$ // of the astd

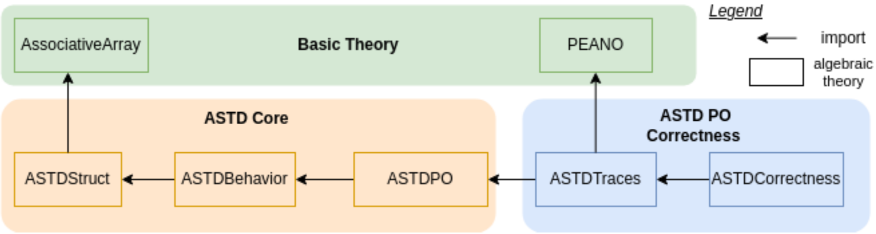
$\wedge POinit(astd, Var \nrightarrow \mathbb{Z}, Var \nrightarrow \mathbb{Z}, (Var \nrightarrow \mathbb{Z}) \lhd id)$
$\wedge POtrans(astd, Var \nrightarrow \mathbb{Z}, \emptyset)$ // when POs hold

$\Rightarrow$
$(\forall i \cdot i \in dom(tr) \Rightarrow InvSpecOnState(astd, tr(i)))$
// every state in the trace satisfies the invariant

Discover ***incomplete*** WD conditions and *minor* **bugs** in the ASTD manual specification (for eg. variable environment).

1. Introduction
   - Motivation
   - Definition
   - Related work
2. An algebraic meta-model for ASTD
   - Background
   - The core meta-theory
   - Two instantiation mechanisms : Deep and Shallow
3. Proof-based reasoning
   - Proof obligations
   - Soundness
4. Conclusion

Overview of EB[ASTD] development



Development effort :

- Definition of 7 **Theories**
- 22 **operators** - 17 **theorems**
- 54 Proof obligations (WD/Thm) for proving theories
- Mostly *interactive* proof but automation is possible

Conclusion

The meta-model provides a solid tool for ASTD by :

- establishing a **proof-based** framework to reason on ASTDs
- offering the possibility of **model-checking** (ProB)
- visualising the **animation** (VisB)
- allowing **automatic** generation of POs
- proving the **soundness** w.r.t. trace semantics

All of this are grounded in a unique tool support Rodin.
The methodology is also suitable for other (state-based) formalism.

Perspectives

- Handling **refinement** in ASTDs
- Temporal, Security properties on ASTDs (ongoing two PhD thesis..)
- Model to Model transformation (EB4EB $\leftrightarrow$ EB[ASTD])

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Appendices

Des figures, listings, documents complets qui peuvent être appuyer pour répondre au question