

# Extending EB4EB for Parameterised Events

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June 10-13, 2023



International Conference  
**ABZ'2025**



# Outline

- 1 Introduction
- 2 Event-B
- 3 The EB4EB reflexive framework
- 4 Revised EB4EB
- 5 Conclusion

# Introduction

## Motivation

**EB4EB Framework**  $\Rightarrow$  Access to Event-B Component + Advanced Reasoning + New PO

**Event Parameter**  $\Rightarrow$  [Missing explicit](#) manipulation in EB4EB

- Local bind to Event
- Each event parameters have different types

## Our proposal

Upgrade EB4EB framework with explicit manipulation of Event-Parameter

- [Polymorphic operator](#) to instantiate at the event level
- [Axiomatic](#) data-type instead of Constructive data-type

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5 Conclusion

# Event-B

**Contexts** (definitions/axioms/theorems) + **Machines** (state and events/inductive invariants)

| Context                   | Machine                               |
|---------------------------|---------------------------------------|
| <b>CONTEXT</b> $C_{tx}$   | <b>MACHINE</b> $M^A$                  |
| <b>SETS</b> $s$           | <b>SEES</b> $C_{tx}$                  |
| <b>CONSTANTS</b> $c$      | <b>VARIABLES</b> $x^A$                |
| <b>AXIOMS</b> $A$         | <b>INVARIANTS</b> $I^A(x^A)$          |
| <b>THEOREMS</b> $T_{ctx}$ | <b>THEOREMS</b> $T_{mch}(x^A)$        |
| <b>END</b>                | <b>VARIANT</b> $V(x^A)$               |
|                           | <b>EVENTS</b>                         |
|                           | INITIALISATION $\triangleq$           |
|                           | ...                                   |
|                           | $evt^A \triangleq$                    |
|                           | <b>ANY</b> $\alpha^A$                 |
|                           | <b>WHERE</b> $G^A(x^A, \alpha^A)$     |
|                           | <b>THEN</b>                           |
|                           | $x^A :  BAP^A(\alpha^A, x^A, x^{A'})$ |
|                           | <b>END</b>                            |
|                           | <b>END</b>                            |

|                                 |                                                                                                                  |
|---------------------------------|------------------------------------------------------------------------------------------------------------------|
| Theorems Context ( $THM\_ctx$ ) | $A \Rightarrow T_{ctx}$                                                                                          |
| Theorems machine ( $THM\_mch$ ) | $A \wedge I^A(x^A) \Rightarrow T_{mch}(x^A)$                                                                     |
| Initialisation (INIT)           | $A \wedge AP^A(\alpha^A, x^{A'}) \Rightarrow I^A(x^{A'})$                                                        |
| Invariant preservation (INV)    | $A \wedge I_A(x^A) \wedge G_A(x^A, \alpha^A) \wedge BAP^A(x^A, \alpha^A, x^{A'}) \Rightarrow I^A(x^{A'})$        |
| Event feasibility (FIS)         | $A \wedge I_A(x^A) \wedge G^A(x^A, \alpha^A) \Rightarrow \exists x^{A'} \cdot BAP^A(x^A, \alpha^A, x^{A'})$      |
| Variant progress (VAR)          | $A \wedge I^A(x^A) \wedge G^A(x^A, \alpha^A) \wedge BAP^A(x^A, \alpha^A, x^{A'}) \Rightarrow V(x^{A'}) < V(x^A)$ |

- **Automatic** generation of POs.
- **Rodin**  $\implies$  Editor + POs generator + automatic/interactive provers

# Event-B Theories

Formalisation of new data-types for Event-B  $\Rightarrow$  Theories

## Theory

```
THEORY Th
IMPORT Th1, ...
TYPE PARAMETERS E, F, ...
DATATYPES
  Type2(E, ...)
  constructors
    cstr1( $p_1:T_1, \dots$ )
OPERATORS
  Op1<nature> ( $p_1:T_1, \dots$ )
    well-definedness  $WD(p_1, \dots)$ 
    direct definition  $D_1$ 
AXIOMATIC DEFINITIONS
  TYPES  $A_1, \dots$ 
  OPERATORS
    AOp2<nature> ( $p_1:T_1, \dots$ ): $T_r$ 
    well-definedness  $WD(p_1, \dots)$ 
  AXIOMS  $A_1, \dots$ 
THEOREMS  $T_1, \dots$ 
END
```

- Algebraic definition for data-types  $\Rightarrow$  **Constructive** definitions and/or **axiomatic** definitions.
- Each operator may be associated with **Well-Definedness (WD)** conditions (partial definitions).
- Relevant proved **theorems**.
- Relevant **inference/rewrite** rules.
- **Tool-support** in Rodin environment.

# Event-B theories: an example

Extend Event-B with **mathematical structures** or **domain knowledge**  
 $\Rightarrow$  *reals, differential equations, kinematics, ontologies...*

**THEORY** Reals **IMPORT** Relations, Rings  
**AXIOMATIC DEFINITIONS**

**TYPES**  $\mathbb{R}$

**OPERATORS**

$Rzero : \mathbb{R},$   
 $Rone : \mathbb{R},$   
 $plus : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R},$   
 $times : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R},$   
 $leq : \mathbb{R} \leftrightarrow \mathbb{R},$   
 ...

**AXIOMS**

$order : order(leq) \wedge total(leq)$   
 $reals : Field(plus, times, Rzero, Rone) \wedge$   
 $integral(times, Rzero) \wedge$   
 $ringCompatible(plus, times, Rzero, Rone, leq)$   
 ...

**MACHINE**  $M$

**VARIABLES**  $x, y,$

**INVARIANTS**

$inv1 : x, y \in \mathbb{R}$   
 $inv2 : Rzero \leq x \wedge x \neq Rzero$   
 $inv3 : y = Rzero \vee y = Rone$

**EVENTS**

...

**Evt**

**WHERE**

$grd1 : x \times times \ y = Rzero$   
 $grd2 : v = Open$

**THEN**  $y := y \ plus \ Rone$

**END**

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# The EB4EB Meta Theory

- **Explicit** manipulation of Event-B features  
 $\implies$  a **constructive data-type** for Event-B machines.

## EventBTheory

```

THEORY EventBTheory
TYPES PARAMETERS STATE, EVENT
DATATYPES
Machine(STATE, EVENT)  $\triangleq$ 
   $\triangleright$  Cons_machine(
    Init: EVENT,
    Progress:  $\mathbb{P}(\text{EVENT})$ ,
    AP:  $\mathbb{P}(\text{STATE})$ ,
    BAP:
       $\mathbb{P}(\text{EVENT} \times (\text{STATE} \times \text{STATE}))$ ,
    Grd:
       $\mathbb{P}(\text{EVENT} \times \text{STATE})$ ,
    Inv:  $\mathbb{P}(\text{STATE})$ ,
    ...)
OPERATORS
...
check_Machine_consistency(m) ...
  
```

- **STATE**: machine states
- **EVENT**: machine events
- **Init**: initialisation event
- **Progress**: progress events
- **AP (After Predicate)**:  
initialisation event action
- **BAP (Before-After Predicate)**:  
actions of progress events
- **Grd**: progress events guards
- **Inv**: machine invariants
- **Check\_Machine\_Consistency**:  
core Event-B POs

## Machine

```

MACHINE M SEES ...
VARIABLES x
INVARIANTS I(x)
...
EVENTS
  INITIALISATION  $\triangleq$ 
    THEN
      x :| AP( $\alpha$ , x)
    END
  EVTi  $\triangleq$ 
    ANY  $\alpha$ 
    WHERE Gi(x,  $\alpha$ )
    THEN
      x :| BAPi( $\alpha$ , x, x)
    END
  ...
  
```

**Check\_Machine\_Consistency(m)**  $\equiv$  PO\_INV(m)  $\wedge$  PO\_THM(m)  $\wedge$  PO\_FIS(m)  $\wedge$  PO\_VAR(m)  $\wedge$  PO\_NAT(m)

# Instantiation of the EB4EB Meta Theory

## Event-B Machines as a FOL expression

- An Event-B machine is seen as a FOL formula (sets, constants, and axioms)

```

CONTEXT Deep
SETS St Ev
CONSTANTS m, ...
AXIOMS
axm1: partition(Ev, ...)
axm2:  $m \in \text{Machine}(\textcolor{brown}{St}, \textcolor{brown}{Ev})$ 
axm3:  $\textcolor{blue}{Event}(m) = \textcolor{brown}{Ev}$ 
axm4:  $\textcolor{brown}{State}(m) = \dots$ 
axm5:  $\textcolor{brown}{Init}(m) = \dots$ 
axm6:  $\textcolor{brown}{Progress}(m) = \{\dots\}$ 
axm7:  $\textcolor{brown}{Thm}(m) = \{\dots\}$ 
axm8:  $\textcolor{blue}{Inv}(m) = \{\dots\}$ 
axm9:  $\textcolor{green}{AP}(m) = \{\dots\}$ 
axm10:  $\textcolor{brown}{Grd}(m) = \{\dots\}$ 
axm11:  $\textcolor{brown}{BAP}(m) = \{\dots\}$ 
axm12:  $\textcolor{brown}{Convergent}(m) = \{\dots\}$ 
axm13:  $\textcolor{brown}{Ordinary}(m) = \{\dots\}$ 
axm14:  $\textcolor{brown}{Variant}(m) = \{\dots\}$ 
THEOREMS
thm1: check_Machine_consistency(m)
END
  
```

- Machine consistency is ensured by discharging the  $\textcolor{blue}{THM\_ctx}$  PO generated for the  $\textcolor{blue}{thm1}$  theorem

$$axm1 \wedge \dots \wedge axm14 \\ \Rightarrow \textcolor{blue}{Check\_Machine\_consistency}(m)$$

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# Event Parameter

## Example

**MACHINE** M

...

**EVENTS**

ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$$

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$$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$$
$$Param(M) \otimes Progress(M) \rightarrow Type$$

# Event Parameter

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**EVENTS**

ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$$\begin{aligned}
 Param(M) &= \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\} \\
 Param(M) &\vDash Progress(M) \rightarrow Type \\
 BAP(M) &\vDash (e \vDash Progress(M) \leftrightarrow \\
 & (Param(M)(e) \times STATE \leftrightarrow STATE))
 \end{aligned}$$

# Event Parameter

## Example

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...

**EVENTS**

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...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$$

$$Param(M) \bowtie Progress(M) \rightarrow \text{Type}$$

$$BAP(M) \bowtie (e \bowtie Progress(M)) \leftrightarrow$$

$$(Param(M)(e) \times STATE \leftrightarrow STATE))$$

Natively Impossible 

# Event Parameter

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$$BAP(M) \vDash (e \vDash Progress(M)) \leftrightarrow$$

$$(\text{Param}(M)(e) \times STATE \leftrightarrow STATE))$$

Natively Impossible 

## Solutions

### 1) Global parameters

**THEORY** EventBTheory

**TYPE PARAMETERS** PARAM, ...

$$PARAM(M) \equiv T_{Par}^{ev1} \times \dots \times T_{Par}^{evn}$$



# Event Parameter

## Example

**MACHINE** M

...

**EVENTS**

ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$$

$$Param(M) \vDash Progress(M) \rightarrow \text{Type}$$

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$$(\text{Param}(M)(e) \times STATE \leftrightarrow STATE))$$

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### 1) Global parameters

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$$PARAM(M) \equiv T_{Par}^{ev1} \times \dots \times T_{Par}^{evn}$$

No local definition of Event

Parameter

# Event Parameter

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ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

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evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$

$Param(M) \vDash Progress(M) \rightarrow \text{Type}$

$BAP(M) \vDash (e \vDash Progress(M)) \leftrightarrow$

$(Param(M)(e) \times STATE \leftrightarrow STATE))$

Natively Impossible 

## Solutions

### 1) Global parameters

**THEORY** EventBTheory

**TYPE PARAMETERS** PARAM, ...

$PARAM(M) \equiv T_{Par}^{ev1} \times \dots \times T_{Par}^{evn}$

No local definition of Event

Parameter

### 2) Axiomatic definition

**AXIOMATIC DATATYPE**

Cons\_mch (EVENT, STATE)

Cons\_Event (PARAM, STATE)

Cons\_Event( $T_{Par}^{ev1}, \dots$ )

# Event Parameter

## Example

**MACHINE** M

...

**EVENTS**

ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$

$$Param(M) = \{ev1 \mapsto T_{Par}^{ev1}\} \cup \dots \cup \{evn \mapsto T_{Par}^{evn}\}$$

$$Param(M) \models Progress(M) \rightarrow \text{Type}$$

$$BAP(M) \models (e \models Progress(M)) \leftrightarrow$$

$$(\text{Param}(M)(e) \times STATE \leftrightarrow STATE))$$

Natively Impossible 

## Solutions

### 1) Global parameters

**THEORY** EventBTheory

**TYPE PARAMETERS** PARAM, ...

$$PARAM(M) \equiv T_{Par}^{ev1} \times \dots \times T_{Par}^{evn}$$

No local definition of Event  
Parameter

### 2) Axiomatic definition

**AXIOMATIC DATATYPE**

Cons\_mch (EVENT, STATE)

Cons\_Event (PARAM, STATE)

Cons\_Event( $T_{Par}^{ev1}, \dots$ )

Solutions presented here

# Inspiration

## Record in classical Event-B

**SETS** `Records, A, B` // `record A {dest1 : A ; dest2 : B}`

**CONSTANTS** `dest1, dest2`

**AXIOMS**

**axm1** `dest1 ∈ Records → A`

**axm1** `dest2 ∈ Records → B`

## Polymorphic operator

**TYPE PARAMETERS** `α, β, ...`

**OPERATORS**

`op1(x : α) ≐ ...`

`op2(y : P(α), z : α × β) ≐ ...`

`op3(w : β × α × Z × ...) ≐ ...`

**CONTEXT** `Ctx`

**AXIOMS**

**axm1** `op1(x : Z × Z) ... // α := Z × Z`

**axm2** `op1(x : P(Z)) ... // α := P(Z)`

**axm3** `op2(y : P(Z × BOOL), z : (Z × BOOL) × BOOL) ... // α := Z × BOOL, β := BOOL`

# Revised Datatype

1) Definition of the record type in Data-Type with Type parameter commun for all the data-type

```
THEORY EvtBTheoPar
TYPE PARAMETERS STATE, EVENT, PARAM
DATATYPES
  Machine(STATE, EVENT) //No PARAM
CONSTRUCTORS
  Cons_machine(
    Event :  $\mathbb{P}(\text{EVENT})$ ,
    State :  $\mathbb{P}(\text{STATE})$ )
```

# Revised Datatype

## 2) Field are definition with Axiomatic Operator

### AXIOMATIC DEFINITIONS OPERATORS

```

Init <expression> (m : Machine(STATE, EVENT)) : EVENT
    // Init ∈ Machine(STATE, EVENT) ↔ EVENT
Progress <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{EVENT})$ 
Param <expression> (m : Machine(STATE, EVENT), e : EVENT,                ) :  $\mathbb{P}(\text{PARAM})$ 
    // Param ∈ Machine(STATE, EVENT) × EVENT ↔  $\mathbb{P}(\text{PARAM})$ 
Grd_par <expression> (m : Machine(STATE, EVENT), e : EVENT,                ) :  $\mathbb{P}(\text{PARAM} \times \text{STATE})$ 
BAP_par <expression>
    (m : Machine(STATE, EVENT), e : EVENT,                ) :  $\mathbb{P}((\text{PARAM} \times \text{STATE}) \times \text{STATE})$ 
Inv <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{STATE})$ 
Thm <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{STATE})$ 
...

```

- Type parameter **Param** is Bound to the definition of the field not to Machine.
- **No axioms** in the theory  $\Rightarrow$  Definition provide in instantiation.

# Revised Datatype

## 2) Field are definition with Axiomatic Operator

### AXIOMATIC DEFINITIONS OPERATORS

```

Init <expression> (m : Machine(STATE, EVENT)) : EVENT
    // Init ∈ Machine(STATE, EVENT) ↔ EVENT
Progress <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{EVENT})$ 
Param <expression> (m : Machine(STATE, EVENT), e : EVENT, p :  $\mathbb{P}(\text{PARAM})$ ) :  $\mathbb{P}(\text{PARAM})$ 
    // Param ∈ Machine(STATE, EVENT) × EVENT ↔  $\mathbb{P}(\text{PARAM})$ 
Grd_par <expression> (m : Machine(STATE, EVENT), e : EVENT, p :  $\mathbb{P}(\text{PARAM})$ ) :  $\mathbb{P}(\text{PARAM} \times \text{STATE})$ 
BAP_par <expression>
    (m : Machine(STATE, EVENT), e : EVENT, p :  $\mathbb{P}(\text{PARAM})$ ) :  $\mathbb{P}((\text{PARAM} \times \text{STATE}) \times \text{STATE})$ 
Inv <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{STATE})$ 
Thm <expression> (m : Machine(STATE, EVENT)) :  $\mathbb{P}(\text{STATE})$ 
...

```

- Type parameter **Param** is Bound to the definition of the field not to Machine.
- **No axioms** in the theory  $\Rightarrow$  Definition provide in instantiation.
- New argument for correct **type checking**.

# New methodology of instantiation for deep medelling

- Classical Event-B

**MACHINE** M

...

**EVENTS**

ev1  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{ev1}$

...

evn  $\triangleq$  **ANY** p **WHEN**  $p \in T_{Par}^{evn}$



# New methodology of instantiation for deep medelling

- Classical Event-B

**MACHINE** M

...

**EVENTS**

$ev1 \triangleq \text{ANY } p \text{ WHEN } p \in T_{Par}^{ev1}$

...

$evn \triangleq \text{ANY } p \text{ WHEN } p \in T_{Par}^{evn}$

- Revised Instantiation

**CONTEXT** EvtInstantiationSchema

**AXIOMS**

...

**AxmParEv.1** :  $Param(m, ev1, T_{Par}^{ev1}) = T_{Par}^{ev1}$

...

**AxmParEv.n** :  $Param(m, evn, T_{Par}^{evn}) = T_{Par}^{evn}$

**AxmParGrd.1** :  $Grd\_par(m, ev1, T_{Par}^{ev1}) = \{par \mapsto s \mid par \in T_{Par}^{ev1} \wedge \dots\}$

...

**AxmParGrd.n** :  $Grd\_par(m, evn, T_{Par}^{evn}) = \{par \mapsto s \mid par \in T_{Par}^{evn} \wedge \dots\}$

**AxmParBAP.1** :  $BAP\_par(m, ev1, T_{Par}^{ev1}) = \{(par \mapsto s) \mapsto sp \mid par \in T_{Par}^{ev1} \wedge \dots\}$

...

**AxmParBAP.n** :  $BAP\_par(m, evn, T_{Par}^{evn}) = \{(par \mapsto s) \mapsto sp \mid par \in T_{Par}^{evn} \wedge \dots\}$

- Param** is instantiated with the **correct type** for each events.
- ⇒ Events components are definition **individually** for each events.

# Revised Proof Obligation

## Case of Invariance

### OPERATORS

```

...
Mch_INV_One_Ev_Par_Def predicate ( m : Machine(STATE, EVENT) , e : EVENT,                ))
    well-definedness e ∈ Progress(m)
    direct definition
         $BAP\_par(m, e)[(Param(m, e, \mathbb{P}(PARAM)) \times Inv(m)) \cap Grd\_par(m, e, \mathbb{P}(PARAM))] \subseteq Inv(m)$ 
Mch_INV_Init predicate ( m : Machine(STATE, EVENT))
    direct definition
         $AP(m) \subseteq Inv(m)$ 
Mch_INV predicate ( m : Machine(STATE, EVENT))
    direct definition
         $Mch\_INV\_Init(m) \wedge (\forall e \cdot e \in Progress(m) \Rightarrow Mch\_INV\_One\_Ev\_Def(m, e,                )))$ 
...

```

# Revised Proof Obligation

## Case of Invariance

### OPERATORS

```

...
Mch_INV_One_Ev_Par_Def predicate (m : Machine(STATE, EVENT), e : EVENT, p :  $\mathbb{P}(\text{PARAM})$ )
  well-definedness e ∈ Progress(m)
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Mch_INV_Init predicate (m : Machine(STATE, EVENT))
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     $AP(m) \subseteq Inv(m)$ 
Mch_INV predicate (m : Machine(STATE, EVENT))
  direct definition
     $Mch\_INV\_Init(m) \wedge (\forall e \cdot e \in Progress(m) \Rightarrow Mch\_INV\_One\_Ev\_Def(m, e, \mathbb{P}(\text{PARAM})))$ 
...

```

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## Case of Invariance

### OPERATORS

```

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Mch_INV_One_Ev_Par_Def predicate (m : Machine(STATE, EVENT), e : EVENT, p :  $\mathbb{P}(\text{PARAM})$ )
  well-definedness e ∈ Progress(m)
  direct definition
     $BAP\_par(m, e)[(Param(m, e, \mathbb{P}(\text{PARAM})) \times Inv(m)) \cap Grd\_par(m, e, \mathbb{P}(\text{PARAM}))] \subseteq Inv(m)$ 
Mch_INV_Init predicate (m : Machine(STATE, EVENT))
  direct definition
     $AP(m) \subseteq Inv(m)$ 
Mch_INV predicate (m : Machine(STATE, EVENT))
  direct definition
     $Mch\_INV\_Init(m) \wedge (\forall e \cdot e \in Progress(m) \Rightarrow Mch\_INV\_One\_Ev\_Def(m, e, \mathbb{P}(\text{PARAM})))$ 
...

```

⚠ Not possible !  $\implies$  require **opacity** on type parameter PARAM

# Revised Proof Obligation

## Opacity Principle

### AXIOMATIC OPERATORS

**Mch**<sub>[P0]</sub>**\_One\_Ev\_Par\_Sig** **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT}$ )

### OPERATORS

...  
**Mch**<sub>[P0]</sub>**\_One\_Ev\_Par\_Def** **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT}, p : \mathbb{P}(\text{PARAM}))$ )

**well-definedness**  $e \in \text{Progress}(m)$

**direct definition**

[update P0 definition with parameter]

**Mch**<sub>[P0]</sub> **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT})$ )

**direct definition**

$\text{Mch}_{[P0]}_{\text{Init}}(m) \wedge (\forall e \cdot e \in \text{Progress}(m) \Rightarrow$  )

### AXIOMS

- Two operators defined :
  - **Def** : PARAM appears and definition provide.
  - **Sig** : PARAM does not appear and no definition.

# Revised Proof Obligation

## Opacity Principle

### AXIOMATIC OPERATORS

$\text{Mch\_}[P0]_{\text{One\_Ev\_Par\_Sig}} \text{ predicate } (m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT})$

### OPERATORS

$\text{Mch\_}[P0]_{\text{One\_Ev\_Par\_Def}} \text{ predicate } (m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT}, p : \mathbb{P}(\text{PARAM}))$

**well-definedness**  $e \in \text{Progress}(m)$

**direct definition**

[update P0 definition with parameter]

$\text{Mch\_}[P0] \text{ predicate } (m : \text{Machine}(\text{STATE}, \text{EVENT}))$

**direct definition**

$\text{Mch\_}[P0]_{\text{Init}}(m) \wedge (\forall e \cdot e \in \text{Progress}(m) \Rightarrow \quad )$

### AXIOMS

**PO[P0]Opacity:**  $\forall m, e \cdot m \in \text{Machine}(\text{STATE}, \text{EVENT}) \wedge e \in \text{Progress}(m)$

$\Rightarrow (\text{Mch\_}[P0]_{\text{One\_Ev\_Par\_Sig}}(m, e) \Leftrightarrow \text{Mch\_}[P0]_{\text{One\_Ev\_Par\_Def}}(m, e, \mathbb{P}(\text{PARAM})))$

- Two operators defined :
  - **Def** : PARAM appears and definition provide.
  - **Sig** : PARAM does not appear and no definition.
- New axiom links the definition **Def** and the opaque operator **Sig**.

# Revised Proof Obligation

## Opacity Principle

### AXIOMATIC OPERATORS

**Mch**<sub>[P0]</sub>**\_One\_Ev\_Par****Sig** **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT}$ )

### OPERATORS

**Mch**<sub>[P0]</sub>**\_One\_Ev\_Par****Def** **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT}), e : \text{EVENT}, p : \mathbb{P}(\text{PARAM}))$ )

**well-definedness**  $e \in \text{Progress}(m)$

**direct definition**

[update P0 definition with parameter]

**Mch**<sub>[P0]</sub> **predicate** ( $m : \text{Machine}(\text{STATE}, \text{EVENT})$ )

**direct definition**

$\text{Mch}_{[P0]}.\text{Init}(m) \wedge (\forall e \cdot e \in \text{Progress}(m) \Rightarrow \text{Mch}_{[P0]}.\text{One\_Ev\_Par}.\text{Sig}(m, e))$

### AXIOMS

**PO**[P0]**Opacity**:  $\forall m, e \cdot m \in \text{Machine}(\text{STATE}, \text{EVENT}) \wedge e \in \text{Progress}(m)$

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- Two operators defined :
  - **Def** : PARAM appears and definition provide.
  - **Sig** : PARAM does not appear and no definition.
- New axiom links the definition **Def** and the opaque operator **Sig**.
- **Sig** can be use for statue the property for all event.

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# Conclusion

## Axiomatic Data-type Comparaision

|              | Axiomatic DT                      | Classical DT           | Record in Ctx      |
|--------------|-----------------------------------|------------------------|--------------------|
| Polymorphism | Heterogeneous Type for field      | Homogeneous type       | No polymorphims    |
| WD           | Mix with WD and external operator | Need external operator | Axioms/invariant   |
| Set          | Partially the field               | Only the field         | Completly          |
| Property     | Operator with opacity             | Operator               | Classical Event-B. |

# Conclusion

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## Limitations

- Equality on the Data-type machine
  - Multiple instationtion of PARAM for same event
- ⇒ No WD on the type instantiation mechanism

# Conclusion

## EB4EB: Proof Obligation and analyses

- Using same principle, **all Native proof obligation** updated with event parameter
- All analyses have been updated, **deadlock, Invariant weakness, Reachability** [RSAD24]
- Revised Clock models
- Bridge case study



Peter Riviere, Neeraj Kumar Singh, Yamine Aït Ameer, and Guillaume Dupont.  
Extending the EB4EB framework with parameterised events.  
[Sc. of Computer Programming, 2024.](#)