An Event-B Sequent Prover in Prolog and ProB

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Recap

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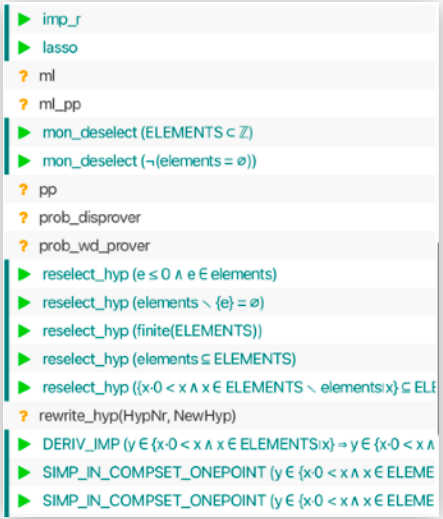
Last Year:

- We started implementing the Rodin proof rules in Prolog
- Initial idea: improve traceability and understanding of proofs in teaching
  - In Rodin, individual proofs steps can be difficult to explore (and understand)
- Turn mathematical definitions into executable Prolog rules that can be used in a prover

https://wiki.event-b.org/index.php/Inference_Rules
https://wiki.event-b.org/index.php/All_Rewrite_Rules

Animate the Proof

- ProB's sequent prover allows to *animate* the proof rules
  - Each PO is a start state
  - Based on the current sequent and the transition predicates, the ProB animator gives the applicable proof rules
  - The state is the current proof sequent (+ continuations)



- Prolog transition for the **`IMP_R`** rule:

```
trans(imp_r, sequent(Hyps, implication(P,Q),Cont),
             sequent(Hyps1,        Q        ,Cont)) :-
    add_hyp(P,Hyps,Hyps1).
```

$$\frac{H, P \vdash Q}{H \vdash P \Rightarrow Q}$$

New Features

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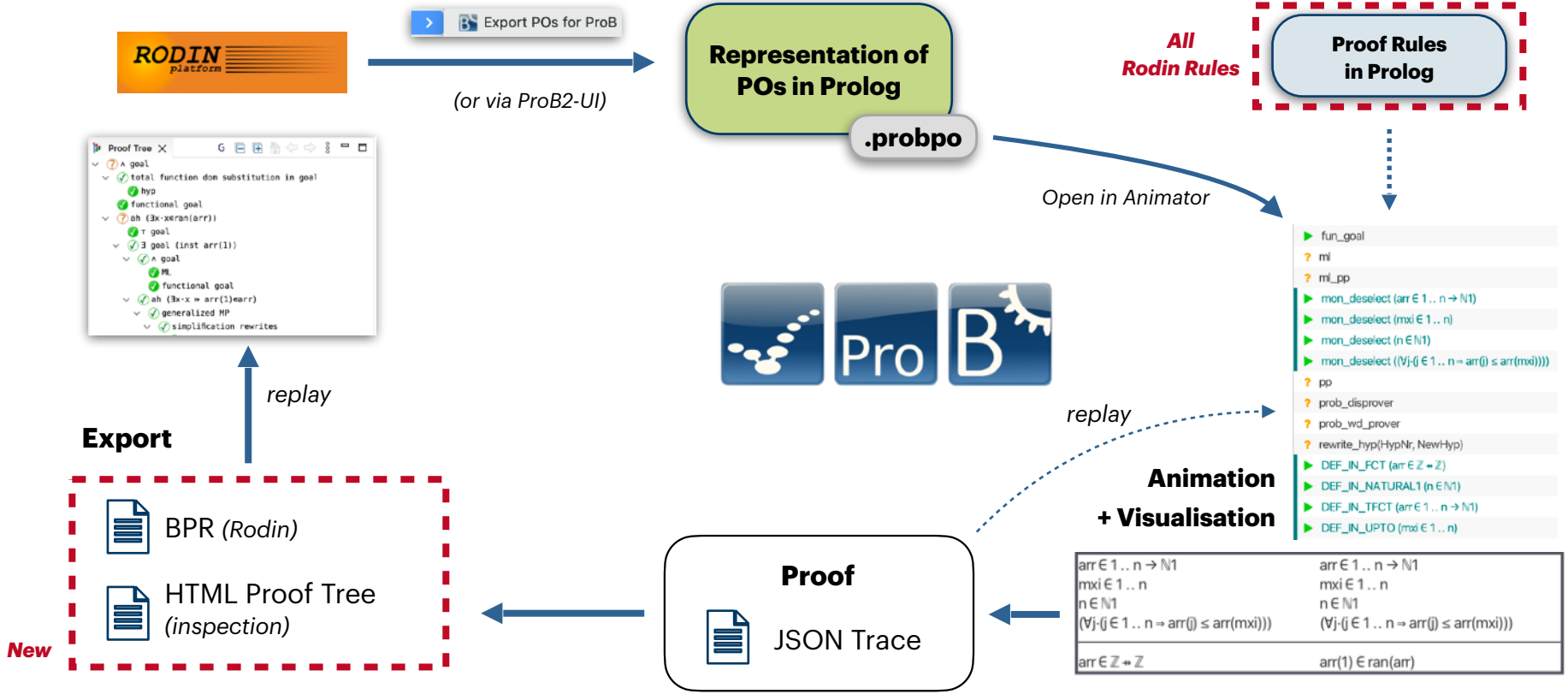
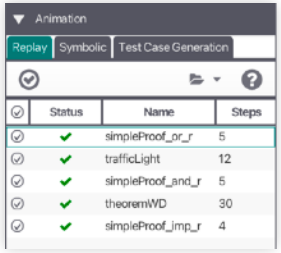
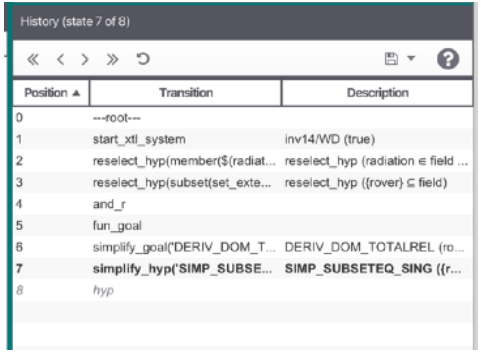
- Allmost all rules implemented (≈ 600) and integrated into the ProB core
- Interactive proving with ProB together with visualisation
- New exports:
  - Interactive HTML proof tree
  - Proof export as Rodin BPR file (experimental)
- First version of an automated prover

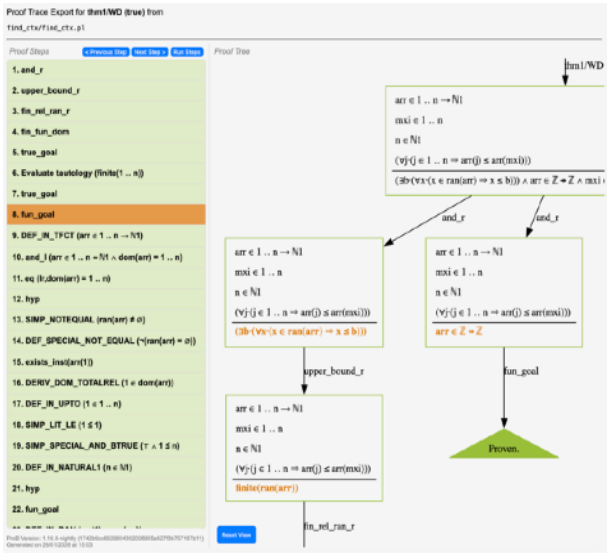
Overview


RODIN platform
Export POs for ProB
(or via ProB2-UI)
Representation of POs in Prolog
.probpo
All Rodin Rules
Proof Rules in Prolog
Open in Animator
Proof Tree
A goal
total function dom substitution in goal
hyp
functional goal
ah (∃x·xeran(arr))
T goal
∃ goal (inst arr(1))
∧ goal
ML
functional goal
ah (∃x·x ↦ arr(1)earr)
generalized MP
simplification rewrites
ProB
replay
Export
BPR (Rodin)
HTML Proof Tree (inspection)
New
Proof
JSON Trace
replay
Animation + Visualisation
fun_goal
mi
mi_pp
mon_deselect (arr ∈ 1 .. n → ℕ1)
mon_deselect (mxi ∈ 1 .. n)
mon_deselect (n ∈ ℕ1)
mon_deselect ((∀j·j ∈ 1 .. n ⇒ arr(j) ≤ arr(mxi)))
pp
prob_disprover
prob_wd_prover
rewrite_hyp(HypNr, NewHyp)
DEF_IN_FCT (arr ∈ ℤ ↔ ℤ)
DEF_IN_NATURAL1 (n ∈ ℕ1)
DEF_IN_TFCT (arr ∈ 1 .. n → ℕ1)
DEF_IN_UPTO (mxi ∈ 1 .. n)
arr ∈ 1 .. n → ℕ1
mxi ∈ 1 .. n
n ∈ ℕ1
(∀j·j ∈ 1 .. n ⇒ arr(j) ≤ arr(mxi))
arr ∈ ℤ ↔ ℤ
arr(1) ∈ ran(arr)

Proof Exports from ProB





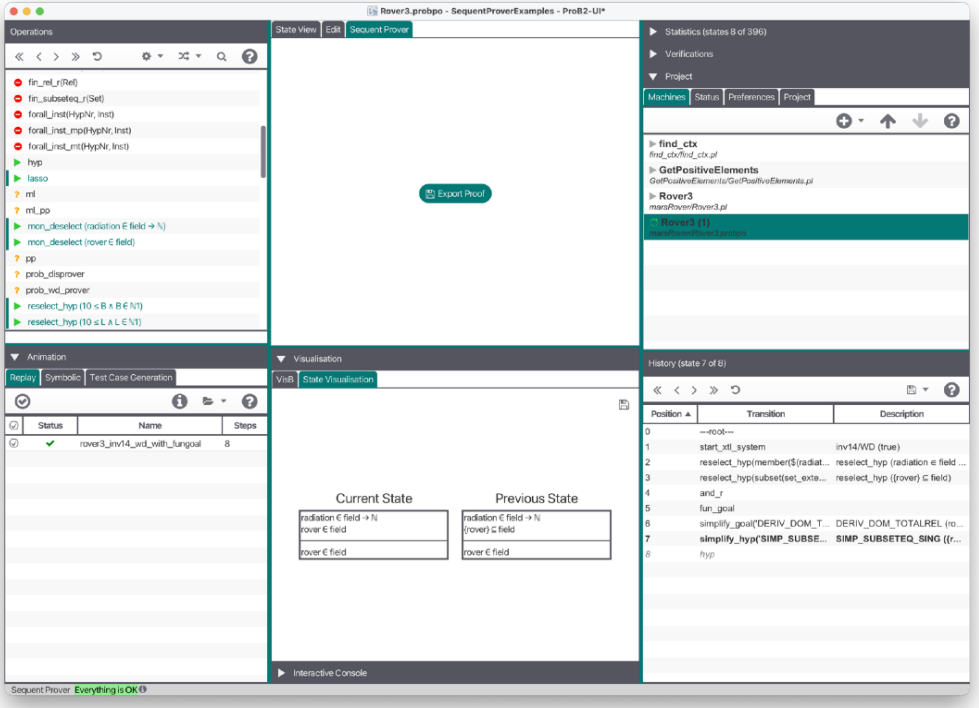


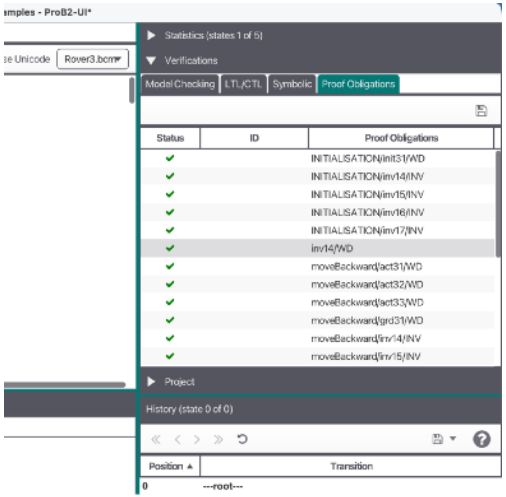
JSON Trace of Proof Steps
replay with ProB

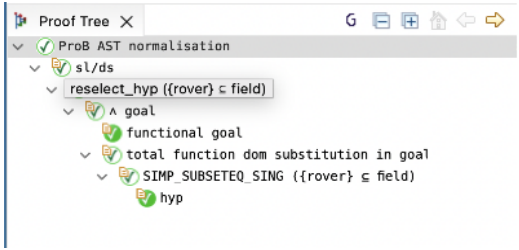
Interactive HTML Document
inspection of the proof tree

BPR Proof File
replay with Rodin
(2nd chain)

Demo: ProB2-UI







Demo: Mars-Rover

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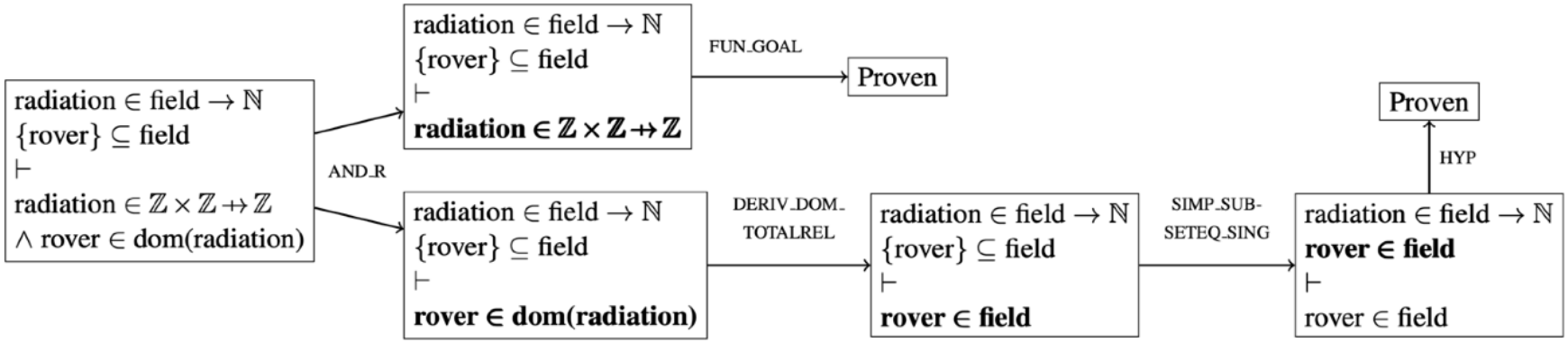
FUN_GOAL $\dfrac{}{\mathbf{H},\ f \in E\ op\ F \ \vdash\ f \in T_1 \nrightarrow T_2}$



AND_R $\dfrac{\mathbf{H} \vdash \mathbf{P} \qquad \mathbf{H} \vdash \mathbf{Q}}{\mathbf{H} \vdash \mathbf{P} \wedge \mathbf{Q}}$

DERIV_DOM_TOTALREL $\mathrm{dom}(r) \mathrel{\widehat{=}} E$

SIMP_SUBSETEQ_SING $\{E\} \subseteq S \mathrel{\widehat{=}} E \in S$

Special Transitions

Rules with User Input:

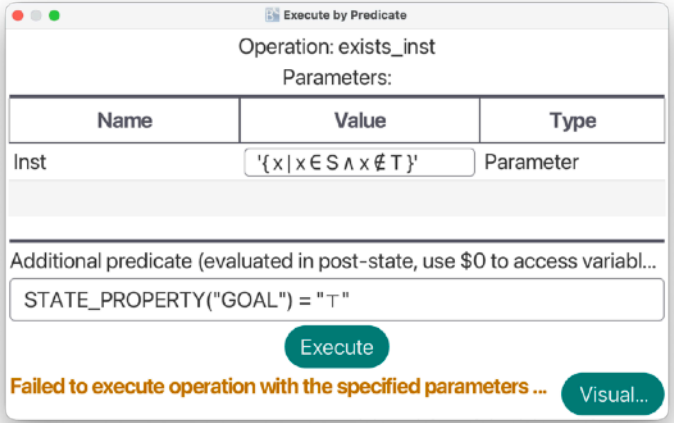
- „execute by predicate“ mechanism
  - transitions arguments provided by the user

Additional Provers:

- Symbolic transitions
  - can only be executed manually
  - call external provers
  - e.g. AtelierB ML/PP, ProB WD/Disprover

EXISTS_INST

$$\frac{\mathbf{H} \vdash WD(E) \qquad \mathbf{H} \vdash \mathbf{P}(E)}{\mathbf{H} \vdash \exists x \cdot \mathbf{P}(x)}$$



Prolog vs Java Encoding of Proof Rules

FIN_SETMINUS_R

$$\frac{\mathbf{H} \vdash \text{finite}(S)}{\mathbf{H} \vdash \text{finite}(S \setminus T)}$$

```
trans_desc_wo_info(
  fin_setminus_r,
  sequent(Hyps,finite(set_subtraction(S,_)),Cont),
  sequent(Hyps,finite(S)                   ,Cont),
  'finite of \'
).
```

```
public class FiniteSetMinus extends EmptyInputReasoner {

        public static final String REASONER_ID = SequentProver.PLUGIN_ID + ".finiteSetMinus";

        @Override
        public String getReasonerID() {
                return REASONER_ID;
        }

        @ProverRule("FIN_SETMINUS_R")
        protected IAntecedent[] getAntecedents(IProverSequent seq) {
                Predicate goal = seq.goal();

                // goal should have the form finite(S \ T)
                if (!Lib.isFinite(goal))
                        return null;
                SimplePredicate sPred = (SimplePredicate) goal;
                if (!Lib.isSetMinus(sPred.getExpression()))
                        return null;

                // There will be 1 antecidents
                IAntecedent[] antecidents = new IAntecedent[1];

                BinaryExpression aExp = (BinaryExpression) sPred.getExpression();

                Expression S = aExp.getLeft();
                final FormulaFactory ff = seq.getFormulaFactory();
                Predicate newGoal = ff.makeSimplePredicate(Predicate.KFINITE, S, null);

                antecidents[0] = ProverFactory.makeAntecedent(newGoal);
                return antecidents;
        }

        protected String getDisplayName() {
                return "finite of \ ";
        }

        @Override
        public IReasonerOutput apply(IProverSequent seq, IReasonerInput input,
                        IProofMonitor pm) {
                IAntecedent[] antecidents = getAntecedents(seq);
                if (antecidents == null)
                        return ProverFactory.reasonerFailure(this, input,
                                        "Inference " + getReasonerID() + " is not applicable");

                // Generate the successful reasoner output
                return ProverFactory.makeProofRule(this, input, seq.goal(),
                                getDisplayName(), antecidents);
        }
}
```

Prolog vs Java Encoding of Proof Rules

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```
simp_rule_with_hyps(domain(R),Dom,'DERIV_DOM_TOTALREL',Hyps) :-
    member(member(R,RType),Hyps), is_rel(RType,total,Dom,_).
simp_rule_with_hyps(range(R),Ran,'DERIV_RAN_SURJREL',Hyps) :-
    member(member(R,RType),Hyps), is_surj(RType,_,Ran).
```

- Java (Rodin): class with more than 200 lines for first rule, second rule not implemented
- Prolog (ProB): add second rule with 1-2 lines of code

Tool	Language	Code for Sequent Prover				Missing Rules	
		Files	LOC	SLOC	Person Years	Rewrite	Inference
Rodin	Java	320	50182	27580	6.51	55/536	16/123
ProB	Prolog	4	4208	3705	0.79	12/536	8/123

The columns SLOC (Source Lines of Code) and the estimated development effort in person years were generated using David A. Wheeler's 'SLOCCount'.

Conclusion and Outlook

- Implemented almost all of Rodin proof rules in Prolog
  - new Sequent Prover mode of ProB
  - HTML: interactive proof tree export — also useful for teaching
  - BPR: proof replay in Rodin — use of ProB prover as second chain
- Ideas for future work:
  - continue and improve automated prover, integrate as Rodin Plugin?
  - PO generation with ProB
  - AI-guided proof
- More ideas, questions?

https://prob.hhu.de/w/index.php?title=Sequent_Prover
https://stups.hhu-hosting.de/models/sequent_prover