

# Composition Operators for Event-B

## CO4EB Rodin plugin

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# Plan

- 1 Introduction
- 2 The Event-B formalisation of composition operators
- 3 The CO4EB plugin

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# The events composition/order

```

MACHINE M0
VARIABLES
  vari
INVARIANTS
  i1 : I(vari)
EVENTS
  Initialisation
  begin
    a1 : init(vari)
  end
  Event Evt0 ≐
  when
    g1 : G0(vari)
  then
    a1 : S0(vari)
  end

```

```

Event Evt1 ≐
  when
    g1 : G1(vari)
  then
    a1 : S1(vari)
  end
Event Evt2 ≐
  when
    g1 : G2(vari)
  then
    a1 : S2(vari)
  end
END

```

## The events order

- is defined by the evaluation of the  $G_i(var_i)$  expressions at each state
- is not explicit in the model

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  end
END

```

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- is defined by the evaluation of the  $G_i(var_i)$  expressions at each state
- is not explicit in the model

# Composition operators

## Operators definition (BNF rules)

```

A ::= A ; A | // sequence
    A [] A | // choice
    A || A | // independent order
    AN | // loop
    ...
    a // atomic
  
```

## Example

$$A_0 ::= A_1 \parallel A_2$$

$$A_1 ::= A_{11} ; A_{12}$$

$$A_0 ::= (A_{11} ; A_{12}) \parallel A_2$$

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# The proposed approach

## Composition operators formalisation

The left hand side part of a BNF rule is modelled by an abstract machine, and the right hand side part by a refinement of this abstract machine.

- $A_i$  action is formalised by  $Evt_i$  event
- $A_0 ::= A_1 \text{ op } A_2$  action is formalised by
  - $M_0$  Event-B Machine with  $Evt_0$  event
  - $M_1$  Event-B refinement with  $Evt_0$ ,  $Evt_1$  and  $Evt_2$  events
  - using an **explicit decreasing variant** for defining the control flow.

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## Composition operators formalisation

The left hand side part of a BNF rule is modelled by an abstract machine, and the right hand side part by a refinement of this abstract machine.

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  - using an **explicit decreasing variant** for defining the control flow.

# The sequence operator : $A_0 ::= A_1 ; A_2$

```

MACHINE  $M_1$ 
VARIABLES
   $var_1$   $varSeq$ 
INVARIANTS
   $inv1 : I(var_1)$ 
   $inv2 : varSeq \in \{0, 1, 2\}$ 
VARIANT
   $varSeq$ 
EVENTS
  Initialisation
  begin
     $act1 : Init(var_1)$ 
     $act2 : varSeq := 2$ 
  end

```

```

Event  $evt_0 \hat{=}$ 
when
   $grd1 : G'(var_1)$ 
   $grd2 : varSeq = 0$ 
then
   $act : A'(var_1)$ 
end

```

```

Event  $evt_1 \hat{=}$ 
Status convergent
when
   $grd1 : G_1(var_1)$ 
   $grd2 : varSeq = 2$ 
then
   $act1 : A_1(var_1)$ 
   $act2 :$ 
     $varSeq := varSeq - 1$ 
end

```

```

Event  $evt_2 \hat{=}$ 
Status convergent
when
   $grd1 : G_2(var_1)$ 
   $grd2 : varSeq = 1$ 
then
   $act1 : A_2(var_1)$ 
   $act2 :$ 
     $varSeq := varSeq - 1$ 
end

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```

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Event  $evt_2 \hat{=}$ 
Status convergent
when
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```

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  end
    
```

# The parallel operator : $A_0 ::= A_1 \parallel A_2$

```

MACHINE  $M_1$ 
VARIABLES
     $var_1$   $varPar_1$   $varPar_2$ 
INVARIANTS
     $inv1 : I(var_1)$ 
     $inv2 : varPar_1 \in \{0, 1\} \wedge varPar_2 \in \{0, 1\}$ 
VARIANT
     $varPar_1 + varPar_2$ 
EVENTS
    Initialisation
    begin
         $act1 : Init(var_1)$ 
         $act2 : varPar_1, varPar_2 := 1, 1$ 
    end
    
```

```

Event  $evt_0 \hat{=}$ 
when
     $grd1 : G'(var_1)$ 
     $grd2 : varPar_1 = 0$ 
     $grd3 : varPar_2 = 0$ 
then
     $act : A'(w)$ 
end
    
```

```

Event  $evt_1 \hat{=}$ 
Status convergent
when
     $grd1 : G_1(var_1)$ 
     $grd2 : varPar_1 = 1$ 
then
     $act1 : A_1(var_1)$ 
     $act2 :$ 
         $varPar_1 := varPar_1 - 1$ 
end
    
```

```

Event  $evt_2 \hat{=}$ 
Status convergent
when
     $grd1 : G_2(var_1)$ 
     $grd2 : varPar_2 = 1$ 
then
     $act1 : A_2(var_1)$ 
     $act2 :$ 
         $varPar_2 := varPar_2 - 1$ 
end
    
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# The choice operator : $A_0 ::= A_1 [] A_2$

MACHINE  $M_1$

VARIABLES

$var_1$  *varCho*

INVARIANTS

inv1 :  $I(var_1)$

inv2 :  $varCho \in \{0, 1, 2\}$

VARIANT

*varCho*

EVENTS

Initialisation

begin

act1 :  $Init(var_1)$

act2 :  $varCho := \{1, 2\}$

end

Event  $evt_0 \hat{=}$

when

grd1 :  $G'(var_1)$

grd2 :  $varCho = 0$

then

act :  $A'(var_1)$

end

Event  $evt_1 \hat{=}$

Status convergent

when

grd1 :  $G_1(var_1)$

grd2 :  $varCho = 1$

then

act1 :  $A_1(var_1)$

act2 :

$varCho := varCho - 1$

end

Event  $evt_2 \hat{=}$

Status convergent

when

grd1 :  $G_2(var_1)$

grd2 :  $varCho = 2$

then

act1 :  $A_2(var_1)$

act2 :

$varCho := varCho - 2$

end

# The choice operator : $A_0 ::= A_1 [] A_2$

```

MACHINE  $M_1$ 
VARIABLES
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VARIANT
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Event  $evt_0 \hat{=}$ 
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  end

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  then
     $act1 : A_1(var_1)$ 
     $act2 : varCho := varCho - 1$ 
  end

```

```

Event  $evt_2 \hat{=}$ 
  Status convergent
  when
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  then
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```

```

Event  $evt_0 \hat{=}$ 
  when
     $grd1 : G'(var_1)$ 
     $grd2 : varCho = 0$ 
  then
     $act : A'(var_1)$ 
  end

```

```

Event  $evt_1 \hat{=}$ 
  Status convergent
  when
     $grd1 : G_1(var_1)$ 
     $grd2 : varCho = 1$ 
  then
     $act1 : A_1(var_1)$ 
     $act2 : varCho := varCho - 1$ 
  end

```

```

Event  $evt_2 \hat{=}$ 
  Status convergent
  when
     $grd1 : G_2(var_1)$ 
     $grd2 : varCho = 2$ 
  then
     $act1 : A_2(var_1)$ 
     $act2 : varCho := varCho - 2$ 
  end

```

# The choice operator : $A_0 ::= A_1 [] A_2$

```

MACHINE  $M_1$ 
VARIABLES
   $var_1$   $varCho$ 
INVARIANTS
   $inv1 : I(var_1)$ 
   $inv2 : varCho \in \{0, 1, 2\}$ 
VARIANT
   $varCho$ 
EVENTS
  Initialisation
  begin
     $act1 : Init(var_1)$ 
     $act2 : varCho := \{1, 2\}$ 
  end

```

```

Event  $evt_0 \hat{=}$ 
  when
     $grd1 : G'(var_1)$ 
     $grd2 : varCho = 0$ 
  then
     $act : A'(var_1)$ 
  end

```

```

Event  $evt_1 \hat{=}$ 
  Status convergent
  when
     $grd1 : G_1(var_1)$ 
     $grd2 : varCho = 1$ 
  then
     $act1 : A_1(var_1)$ 
     $act2 : varCho := varCho - 1$ 
  end

```

```

Event  $evt_2 \hat{=}$ 
  Status convergent
  when
     $grd1 : G_2(var_1)$ 
     $grd2 : varCho = 2$ 
  then
     $act1 : A_2(var_1)$ 
     $act2 : varCho := varCho - 2$ 
  end

```

# The loop operator : $A_0 ::= A_1^n$

```

MACHINE  $M_1$ 
VARIABLES
     $var_1$   $varLoop$ 
INVARIANTS
     $inv1 : I(var_1)$ 
     $inv2 : varLoop \in \mathbb{N}$ 
VARIANT
     $varLoop$ 
EVENTS
    
```

```

Initialisation
begin
    act1 :  $Init(var_1)$ 
    act2 :  $varLoop := 1$ 
end
    
```

```

Event  $evt_0 \hat{=}$ 
when
     $grd1 : G'(var_1)$ 
     $grd2 : varLoop = 0$ 
then
    act :  $A'(var_1)$ 
end
    
```

```

Event  $evt_1 \hat{=}$ 
Status convergent
when
     $grd1 : G_1(var_1)$ 
     $grd2 : varLoop \neq 0$ 
then
    act1 :  $A_1(var_1)$ 
    act2 :
         $varLoop := varLoop - 1$ 
end
    
```



# The loop operator : $A_0 ::= A_1^n$

```

MACHINE  $M_1$ 
VARIABLES
   $var_1$   $varLoop$ 
INVARIANTS
   $inv1$  :  $I(var_1)$ 
   $inv2$  :  $varLoop \in \mathbb{N}$ 
VARIANT
   $varLoop$ 
EVENTS
  
```

```

Initialisation
  begin
    act1 :  $Init(var_1)$ 
    act2 :  $varLoop := 1$ 
  end
  
```

```

Event  $evt_0 \hat{=}$ 
  when
     $grd1$  :  $G'(var_1)$ 
     $grd2$  :  $varLoop = 0$ 
  then
    act :  $A'(var_1)$ 
  end
  
```

```

Event  $evt_1 \hat{=}$ 
  Status convergent
  when
     $grd1$  :  $G_1(var_1)$ 
     $grd2$  :  $varLoop \neq 0$ 
  then
    act1 :  $A_1(var_1)$ 
    act2 :
       $varLoop := varLoop - 1$ 
  end
  
```

# The loop operator : $A_0 ::= A_1^n$

```

MACHINE  $M_1$ 
VARIABLES
   $var_1$   $varLoop$ 
INVARIANTS
   $inv1$  :  $I(var_1)$ 
   $inv2$  :  $varLoop \in \mathbb{N}$ 
VARIANT
   $varLoop$ 
EVENTS
  
```

```

Initialisation
begin
  act1 :  $Init(var_1)$ 
  act2 :  $varLoop := \mathbb{N}1$ 
end
  
```

```

Event  $evt_0 \hat{=}$ 
when
   $grd1$  :  $G'(var_1)$ 
   $grd2$  :  $varLoop = 0$ 
then
  act :  $A'(var_1)$ 
end
  
```

```

Event  $evt_1 \hat{=}$ 
Status convergent
when
   $grd1$  :  $G_1(var_1)$ 
   $grd2$  :  $varLoop \neq 0$ 
then
  act1 :  $A_1(var_1)$ 
  act2 :
     $varLoop := varLoop - 1$ 
end
  
```

# The loop operator : $A_0 ::= A_1^n$

```

MACHINE  $M_1$ 
VARIABLES
     $var_1$   $varLoop$ 
INVARIANTS
     $inv1 : I(var_1)$ 
     $inv2 : varLoop \in \mathbb{N}$ 
VARIANT
     $varLoop$ 
EVENTS
    
```

```

Initialisation
begin
    act1 :  $Init(var_1)$ 
    act2 :  $varLoop : \in \mathbb{N}1$ 
end
    
```

```

Event  $evt_0 \hat{=}$ 
when
     $grd1 : G'(var_1)$ 
     $grd2 : varLoop = 0$ 
then
    act :  $A'(var_1)$ 
end
    
```

```

Event  $evt_1 \hat{=}$ 
Status convergent
when
     $grd1 : G_1(var_1)$ 
     $grd2 : varLoop \neq 0$ 
then
    act1 :  $A_1(var_1)$ 
    act2 :
         $varLoop := varLoop - 1$ 
end
    
```

# The proposed approach

## example

$A_0 ::= A_1 \parallel A_2$

$A_1 ::= A_{11}; A_{12}$

$A_0 ::= (A_{11}; A_{12}) \parallel A_2$

## Machine $M_0 \hat{=} A_0$

```
MACHINE M0
VARIABLES
  vari
INVARIANTS
  i : I(vari)
EVENTS
  Initialisation
    begin
      a1 : init(vari)
    end
  Event Evt0  $\hat{=}$ 
    when
      g : G0(vari)
    then
      a : S0(vari)
    end
END
```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  var_j varPar1 varPar2
INVARIANTS
  i1 : J(var_j, var_j)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(var_j)
    s1 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'_0(var_j)
    s2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'_0(var_j)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G_1(var_j)
    s2 : varPar1 = 1
  then
    a1 : S_1(var_j)
    s2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G_2(var_j)
    s1 : varPar2 = 1
  then
    a1 : S_2(var_j)
    s1 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  var_j varPar1 varPar2
INVARIANTS
  i1 : J(var_j, var_j)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(var_j)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'_0(var_j)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'_0(var_j)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G_1(var_j)
    g2 : varPar1 = 1
  then
    a1 : S_1(var_j)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G_2(var_j)
    g2 : varPar2 = 1
  then
    a1 : S_2(var_j)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  var_j varPar1 varPar2
INVARIANTS
  i1 : J(var_j, var_j)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(var_j)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(var_j)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(var_j)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(var_j)
    g2 : varPar1 = 1
  then
    a1 : S1(var_j)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(var_j)
    g2 : varPar2 = 1
  then
    a1 : S2(var_j)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  varj varPar1 varPar2
INVARIANTS
  i1 : J(varj, vari)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(varj)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(varj)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(varj)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(varj)
    g2 : varPar1 = 1
  then
    a1 : S1(varj)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(varj)
    g2 : varPar2 = 1
  then
    a1 : S2(varj)
    a2 : varPar2 := varPar2 - 1
  end
END

```



# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  varj varPar1 varPar2
INVARIANTS
  i1 : J(varj, varj)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(varj)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(varj)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(varj)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(varj)
    g2 : varPar1 = 1
  then
    a1 : S1(varj)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(varj)
    g2 : varPar2 = 1
  then
    a1 : S2(varj)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  varj varPar1 varPar2
INVARIANTS
  i1 : J(varj, varj)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(varj)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(varj)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(varj)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(varj)
    g2 : varPar1 = 1
  then
    a1 : S1(varj)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(varj)
    g2 : varPar2 = 1
  then
    a1 : S2(varj)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  varj varPar1 varPar2
INVARIANTS
  i1 : J(varj, varj)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(varj)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(varj)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(varj)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(varj)
    g2 : varPar1 = 1
  then
    a1 : S1(varj)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(varj)
    g2 : varPar2 = 1
  then
    a1 : S2(varj)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

Machine  $M_1 \hat{=} A_1 \parallel A_2$ ,  
variant =  $\text{varPar}_1 + \text{varPar}_2$

```

MACHINE M1
REFINES M0
VARIABLES
  varj varPar1 varPar2
INVARIANTS
  i1 : J(varj, varj)
  i2 : varPar1 ∈ {0, 1} ∧ varPar2 ∈ {0, 1}
VARIANT
  varPar1 + varPar2
EVENTS
  Initialisation
  begin
    a1 : init(varj)
    a2 : varPar1, varPar2 := 1, 1
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G'0(varj)
    g2 : varPar1 = 0 ∧ varPar2 = 0
  then
    a : S'0(varj)
  end

```

```

Event Evt1 ≐
  Status convergent
  when
    g1 : G1(varj)
    g2 : varPar1 = 1
  then
    a1 : S1(varj)
    a2 : varPar1 := varPar1 - 1
  end
Event Evt2 ≐
  Status convergent
  when
    g1 : G2(varj)
    g2 : varPar2 = 1
  then
    a1 : S2(varj)
    a2 : varPar2 := varPar2 - 1
  end
END

```

# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

```

MACHINE M2
REFINES M1
VARIABLES
  vark varSeq
INVARIANTS
  i1 : K(vark, varj, vari)
  i2 : varSeq ∈ {0, 1, 2}
VARIANT
  varSeq
EVENTS
  Initialisation
  begin
    a1 : init(vark)
    a2 : varSeq := 2
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G0''(vark)
    g2 : varPar1 = 0
    g3 : varPar2 = 0
  then
    a : S0''(vark)
  end

```

```

Event Evt11 ≐
Status convergent
when
  g1 : G11(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 2
then
  a1 : S11(vark)
  a2 : varSeq := varSeq - 1
end
Event Evt12 ≐
Status convergent
when
  g1 : G12(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 1
then
  a1 : S12(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt1 ≐
refines Evt1
when
  g1 : G'1(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 0
then
  a1 : S'1(vark)
  a2 : varPar1 := varPar1 - 1
end
Event Evt2 ≐
refines Evt2
when
  g1 : G'2(vark)
  g2 : varPar2 = 1
then
  a1 : S'2(vark)
  a2 : varPar2 := varPar2 - 1
end
END

```

# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

```

MACHINE M2
REFINES M1
VARIABLES
  vark varSeq
INVARIANTS
  i1 : K(vark, varj, vari)
  i2 : varSeq ∈ {0, 1, 2}
VARIANT
  varSeq
EVENTS
  Initialisation
  begin
    a1 : init(vark)
    a2 : varSeq := 2
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G0''(vark)
    g2 : varPar1 = 0
    g3 : varPar2 = 0
  then
    a : S0''(vark)
  end
    
```

```

Event Evt11 ≐
Status convergent
when
  g1 : G11(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 2
then
  a1 : S11(vark)
  a2 : varSeq := varSeq - 1
end
Event Evt12 ≐
Status convergent
when
  g1 : G12(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 1
then
  a1 : S12(vark)
  a2 : varSeq := varSeq - 1
end
    
```

```

Event Evt1 ≐
refines Evt1
when
  g1 : G'1(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 0
then
  a1 : S'1(vark)
  a2 : varPar1 := varPar1 - 1
end
Event Evt2 ≐
refines Evt2
when
  g1 : G'2(vark)
  g2 : varPar2 = 1
then
  a1 : S'2(vark)
  a2 : varPar2 := varPar2 - 1
end
END
    
```

# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

```

MACHINE M2
REFINES M1
VARIABLES
  vark varSeq
INVARIANTS
  i1 : K(vark, varj, vari)
  i2 : varSeq ∈ {0, 1, 2}
VARIANT
  varSeq
EVENTS
  Initialisation
  begin
    a1 : init(vark)
    a2 : varSeq := 2
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G0''(vark)
    g2 : varPar1 = 0
    g3 : varPar2 = 0
  then
    a : S0''(vark)
  end

```

```

Event Evt11 ≐
Status convergent
when
  g1 : G11(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 2
then
  a1 : S11(vark)
  a2 : varSeq := varSeq - 1
end
Event Evt12 ≐
Status convergent
when
  g1 : G12(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 1
then
  a1 : S12(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt1 ≐
refines Evt1
when
  g1 : G'1(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 0
then
  a1 : S'1(vark)
  a2 : varPar1 := varPar1 - 1
end
Event Evt2 ≐
refines Evt2
when
  g1 : G'2(vark)
  g2 : varPar2 = 1
then
  a1 : S'2(vark)
  a2 : varPar2 := varPar2 - 1
end
END

```

# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

```

MACHINE M2
REFINES M1
VARIABLES
  vark varSeq
INVARIANTS
  i1 : K(vark, varj, vari)
  i2 : varSeq ∈ {0, 1, 2}
VARIANT
  varSeq
EVENTS
  Initialisation
  begin
    a1 : init(vark)
    a2 : varSeq := 2
  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G0''(vark)
    g2 : varPar1 = 0
    g3 : varPar2 = 0
  then
    a : S0''(vark)
  end

```

```

Event Evt11 ≐
Status convergent
when
  g1 : G11(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 2
then
  a1 : S11(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt12 ≐
Status convergent
when
  g1 : G12(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 1
then
  a1 : S12(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt1 ≐
refines Evt1
when
  g1 : G'1(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 0
then
  a1 : S'1(vark)
  a2 : varPar1 := varPar1 - 1
end
Event Evt2 ≐
refines Evt2
when
  g1 : G'2(vark)
  g2 : varPar2 = 1
then
  a1 : S'2(vark)
  a2 : varPar2 := varPar2 - 1
end
END

```



# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

```

MACHINE M2
REFINES M1
VARIABLES
  vark varSeq
INVARIANTS
  i1 : K(vark, varj, vari)
  i2 : varSeq ∈ {0, 1, 2}
VARIANT
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EVENTS
  Initialisation
  begin
    a1 : init(vark)
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  end
  Event Evt0 ≐
  refines Evt0
  when
    g1 : G0''(vark)
    g2 : varPar1 = 0
    g3 : varPar2 = 0
  then
    a : S0''(vark)
  end

```

```

Event Evt11 ≐
Status convergent
when
  g1 : G11(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 2
then
  a1 : S11(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt12 ≐
Status convergent
when
  g1 : G12(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 1
then
  a1 : S12(vark)
  a2 : varSeq := varSeq - 1
end

```

```

Event Evt1 ≐
refines Evt1
when
  g1 : G'1(vark)
  g2 : varPar1 = 1
  g3 : varSeq = 0
then
  a1 : S'1(vark)
  a2 : varPar1 := varPar1 - 1
end
Event Evt2 ≐
refines Evt2
when
  g1 : G'2(vark)
  g2 : varPar2 = 1
then
  a1 : S'2(vark)
  a2 : varPar2 := varPar2 - 1
end
END

```

# The proposed approach

**Machine  $M_2 \hat{=} (A_{11} ; A_{12}) \parallel A_2$**   
**variant = (varPar<sub>1</sub> + varPar<sub>2</sub>) + varSeq**

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END
  
```

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```

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END
    
```

# The proposed approach

## Interleaving semantic of Event-B

- Machine  $M_1$

- $(A_1 ; A_2) \sqsubseteq (A_1 \parallel A_2)$
- $(A_2 ; A_1) \sqsubseteq (A_1 \parallel A_2)$

- Machine  $M_2$

- $(A_{11} ; A_{12} ; A_2) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$
- $(A_{11} ; A_2 ; A_{12}) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$
- $(A_2 ; A_{11} ; A_{12}) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$



# The proposed approach

## Interleaving semantic of Event-B

- Machine  $M_1$ 
  - $(A_1 ; A_2) \sqsubseteq (A_1 \parallel A_2)$
  - $(A_2 ; A_1) \sqsubseteq (A_1 \parallel A_2)$
- Machine  $M_2$ 
  - $(A_{11} ; A_{12} ; A_2) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$
  - $(A_{11} ; A_2 ; A_{12}) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$
  - $(A_2 ; A_{11} ; A_{12}) \sqsubseteq ((A_{11} ; A_{12}) \parallel A_2)$

# Plan

- 1 Introduction
- 2 The Event-B formalisation of composition operators
- 3 The CO4EB plugin

# The CO4EB plugin/Conclusion

## Update site

`idir.aitsadoune.free.fr/tools/updatesite`

## Video demo

`www.youtube.com/watch?v=oQwsu8CQF0w`