

# Building Event-B Interlocking Theories

## Lessons Learned Using the Theory Plug-in

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PLUS HAUT  
ET PLUS PROCHE  
LE FONDS EUROPÉEN DE DÉVELOPPEMENT RÉGIONAL  
ET LA WALLONIE INVESTISSENT DANS VOTRE AVENIR



Centre d'Excellence en **Technologies** de  
l'**Information** et de la **Communication**

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# Who Am I ?

Yet another formal methods research engineer...

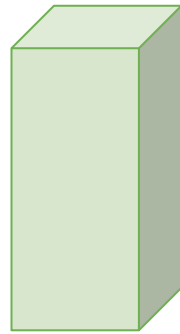
...with a bit of interest in the event-b method.

Before @ Systere

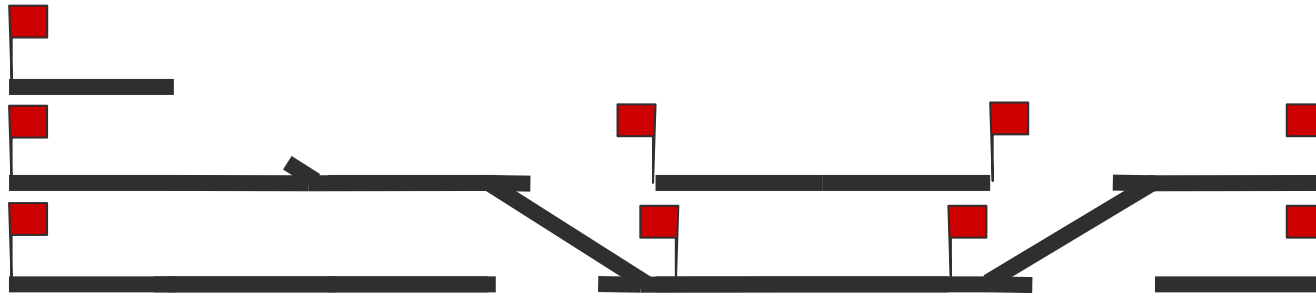
Coded for the SMT Solvers Plug-in of Rodin

Nowadays @ **CETIC** : *Technology Transfert*  
Experimenting with the Theory Plug-in of Rodin

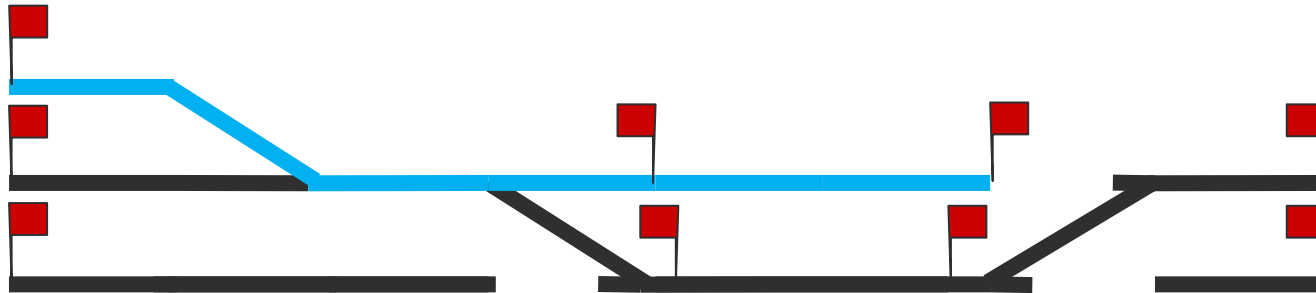
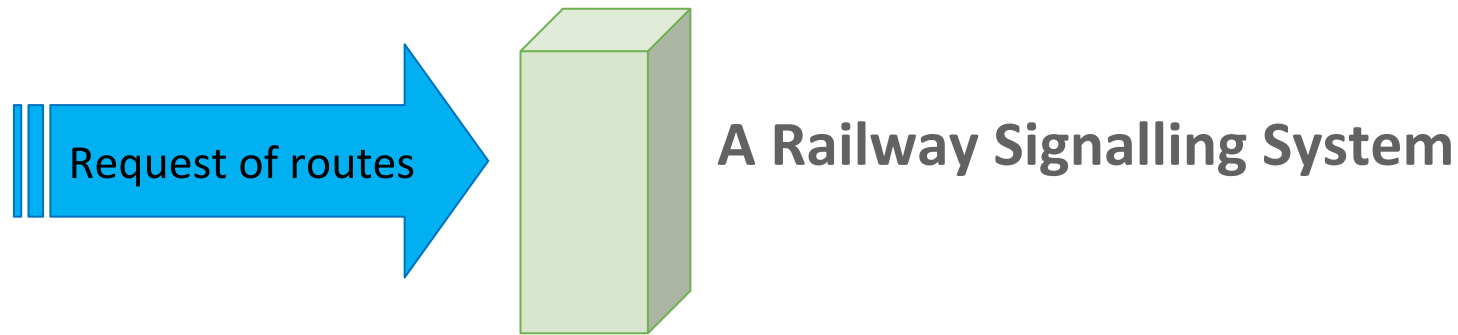
# What is an Interlocking ?



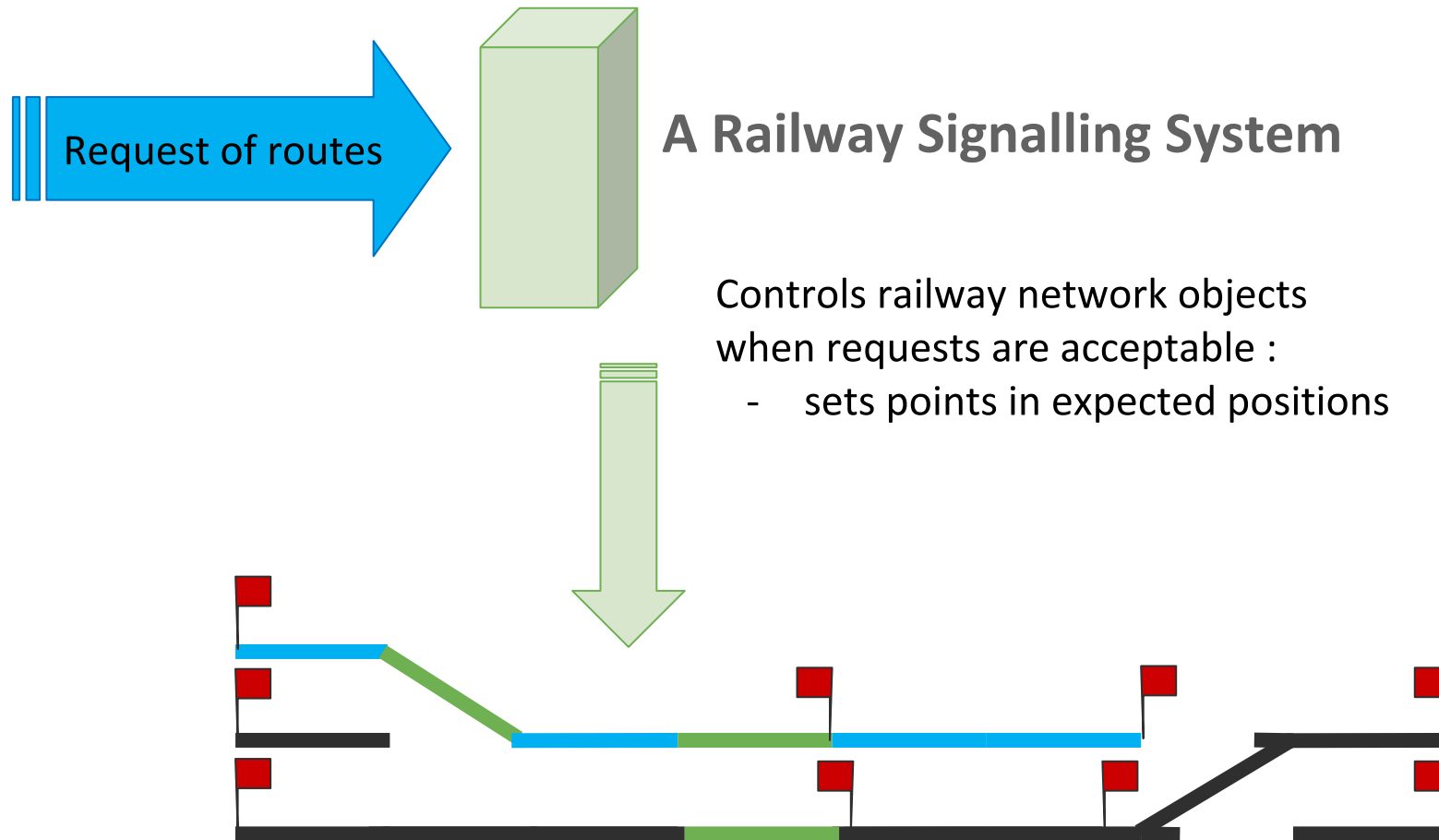
## A Railway Signalling System



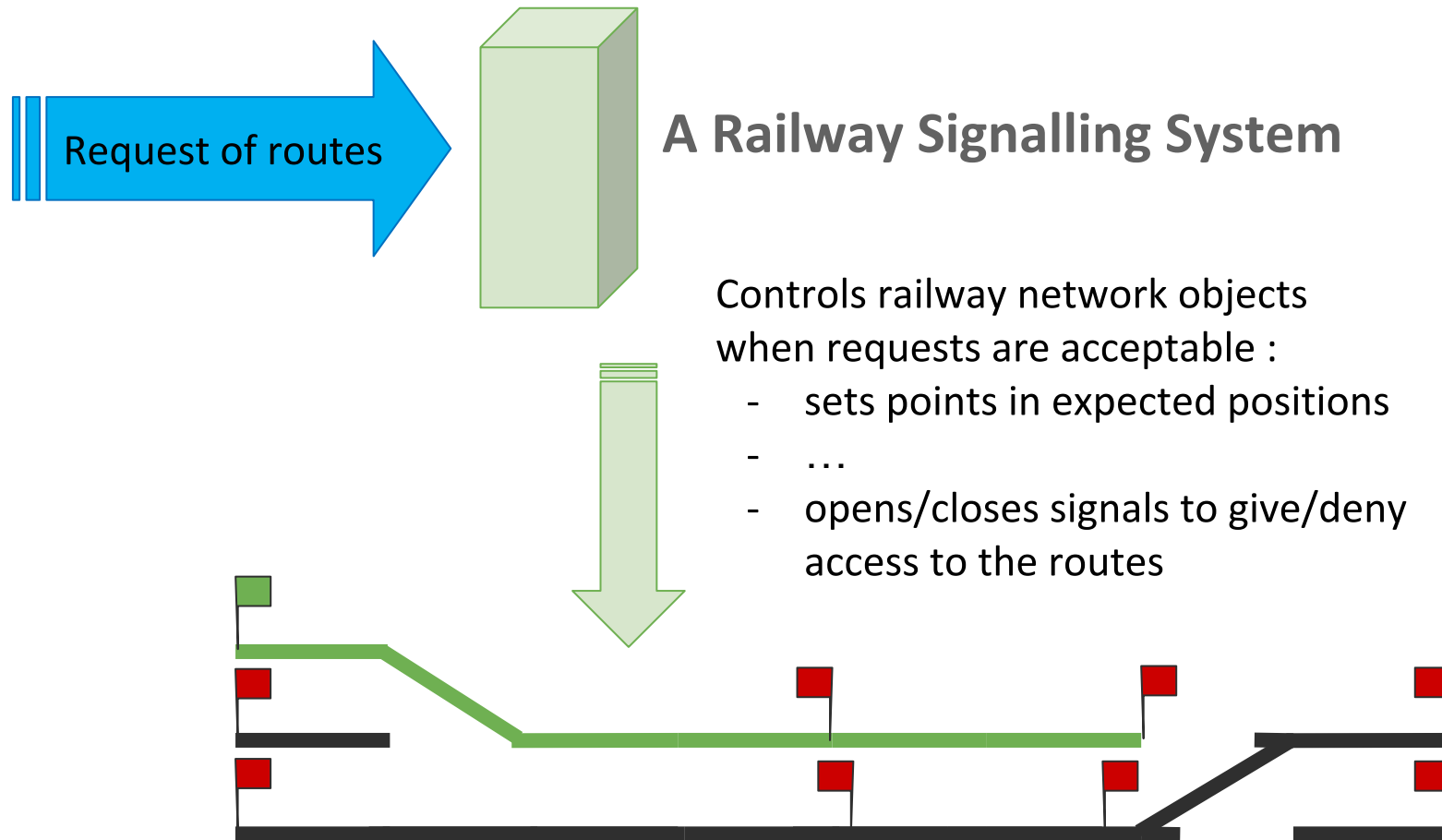
# What is an Interlocking ?



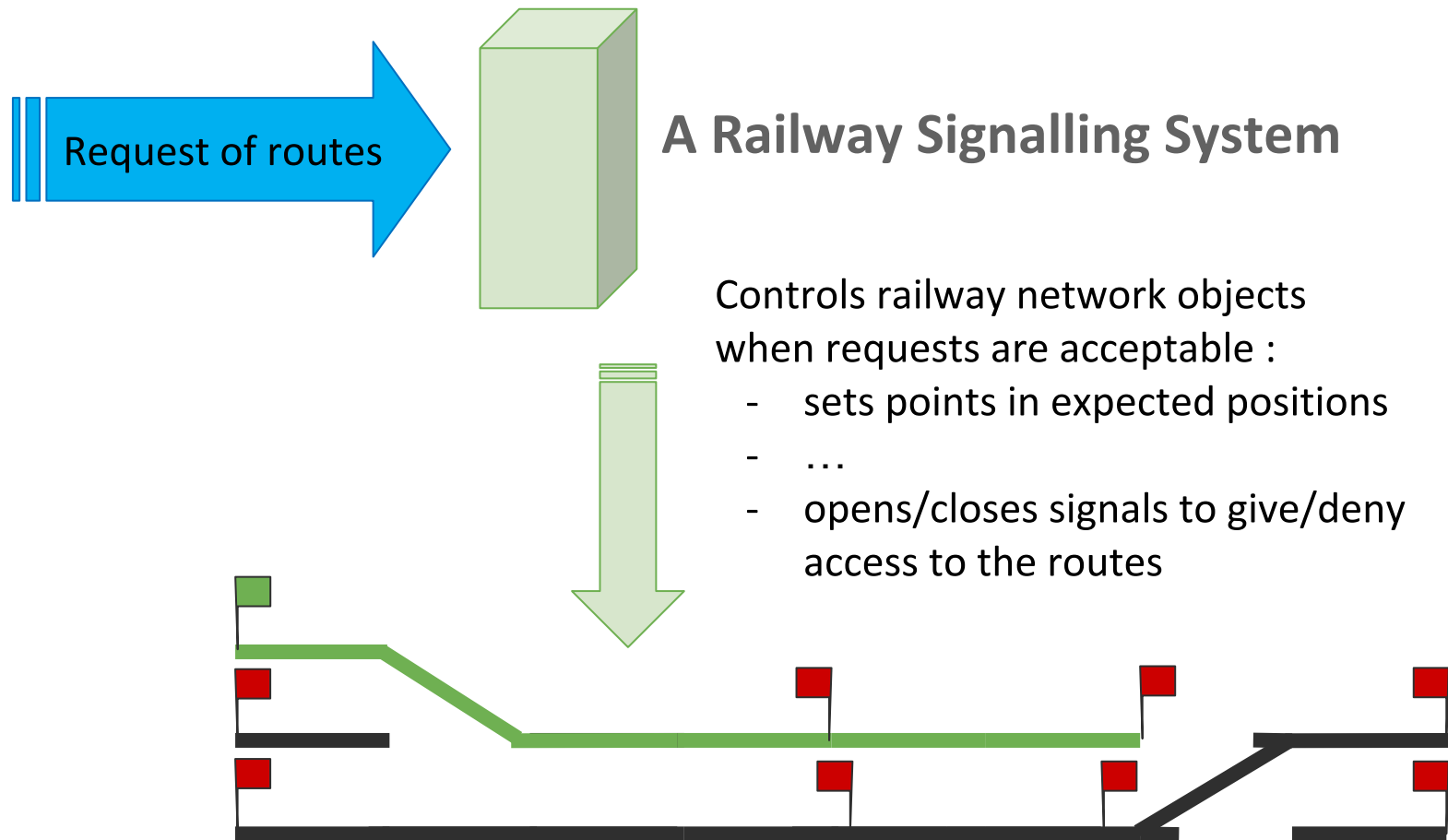
# What is an Interlocking ?



# What is an Interlocking ?



# What is an Interlocking ?



**Trains move on the network without colliding with each others  
nor derailling...**

# Who Makes Interlockings ?

Signalling engineers...

- possess the historical knowledge
- incrementally design new interlockings
- directly produce specific code  
based on generic programming rules
- *validate by reviewing and testing*



# Who Makes Interlockings ?

Signalling engineers...

- possess the historical knowledge
- incrementally design new interlockings
- directly produce specific code based on generic programming rules
- *validate by reviewing and testing*

What about mathematical  
*proof of safety* ?

- are generally *not* fluent in formal methods
- are especially *not* event-b lovers



# Outline

## How to Bring These Worlds Together *Use the Theory Plug-in of Rodin*<sup>1</sup>

“Cover up those mathematical expressions which I cannot endure...”<sup>2</sup>

Defining Interlocking Theories

Reducing the Proof Effort

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<sup>1</sup>Modeling a Safe Interlocking Using the Event-B Theory Plug-in, M-T. Khuu, L. Voisin and F. Mejia

<sup>2</sup>(almost) Molière’s Tartuffe

# Covering Mathematical Expressions

# The Famous Train Example First Context

- axm1:**  $\text{blocks\_routes} \in \text{BLOCKS} \leftrightarrow \text{ROUTES}$
- axm2:**  $\text{dom}(\text{blocks\_routes}) = \text{BLOCKS}$
- axm3:**  $\text{ran}(\text{blocks\_routes}) = \text{ROUTES}$
- axm4:**  $\text{next} \in \text{ROUTES} \rightarrow (\text{BLOCKS} \sqcap \text{BLOCKS})$
- axm5:**  $\text{fst} \in \text{ROUTES} \rightarrow \text{BLOCKS}$
- axm6:**  $\text{last} \in \text{ROUTES} \rightarrow \text{BLOCKS}$
- axm7:**  $\text{fst}^{\sim} \subseteq \text{blocks\_routes}$
- axm8:**  $\text{last}^{\sim} \subseteq \text{blocks\_routes}$
- axm9:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow \text{fst}(r) \neq \text{last}(r)$
- axm10:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow \text{next}(r) \in \text{blocks\_routes}^{\sim}[\{r\}] \setminus \{\text{last}(r)\} \sqcap \text{blocks\_routes}^{\sim}[\{r\}] \setminus \{\text{fst}(r)\}$
- axm11:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow (\forall S \cdot S \subseteq \text{next}(r)[S] \Rightarrow S = \emptyset)$
- axm12:**  $\forall r_1, r_2 \cdot r_1 \in \text{ROUTES} \wedge r_2 \in \text{ROUTES} \wedge r_1 \neq r_2$   
 $\Rightarrow \text{fst}(r_1) \notin \text{blocks\_routes}^{\sim}[\{r_2\}] \setminus \{\text{fst}(r_2), \text{last}(r_2)\}$
- axm13:**  $\forall r_1, r_2 \cdot r_1 \in \text{ROUTES} \wedge r_2 \in \text{ROUTES} \wedge r_1 \neq r_2$   
 $\Rightarrow \text{last}(r_1) \notin \text{blocks\_routes}^{\sim}[\{r_2\}] \setminus \{\text{fst}(r_2), \text{last}(r_2)\}$

# The Famous Train Example First Context

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**axm3:**  $\text{ran}(\text{blocks\_routes}) = \text{ROUTES}$

Objects definitions

**axm4:**  $\text{next} \in \text{ROUTES} \rightarrow (\text{BLOCKS} \sqcap \text{BLOCKS})$

**axm5:**  $\text{fst} \in \text{ROUTES} \rightarrow \text{BLOCKS}$

**axm6:**  $\text{last} \in \text{ROUTES} \rightarrow \text{BLOCKS}$

**axm7:**  $\text{fst}^{\sim} \subseteq \text{blocks\_routes}$

**axm8:**  $\text{last}^{\sim} \subseteq \text{blocks\_routes}$

Operators for manipulating the objects

Expected properties of the objects

**axm9:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow \text{fst}(r) \neq \text{last}(r)$

**axm10:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow \text{next}(r) \in \text{blocks\_routes}^{\sim}[\{r\}] \setminus \{\text{last}(r)\} \sqcap \text{blocks\_routes}^{\sim}[\{r\}] \setminus \{\text{fst}(r)\}$

**axm11:**  $\forall r \cdot r \in \text{ROUTES} \Rightarrow (\forall S \cdot S \subseteq \text{next}(r)[S] \Rightarrow S = \emptyset)$

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# What Is Our Goal ?

Be as close as possible to the signaling engineers usage:

- Implicit objects definitions (blocks, routes, signals...)
- Implicit operators definitions (reserve, lock, open...)
- Rich DSL
- Implicit objects basic properties

Just model *this* interlocking.

# The Famous Train Example First Context

- axm1:**  $\text{blocks\_routes} \in \text{BLOCKS} \leftrightarrow \text{ROUTES}$
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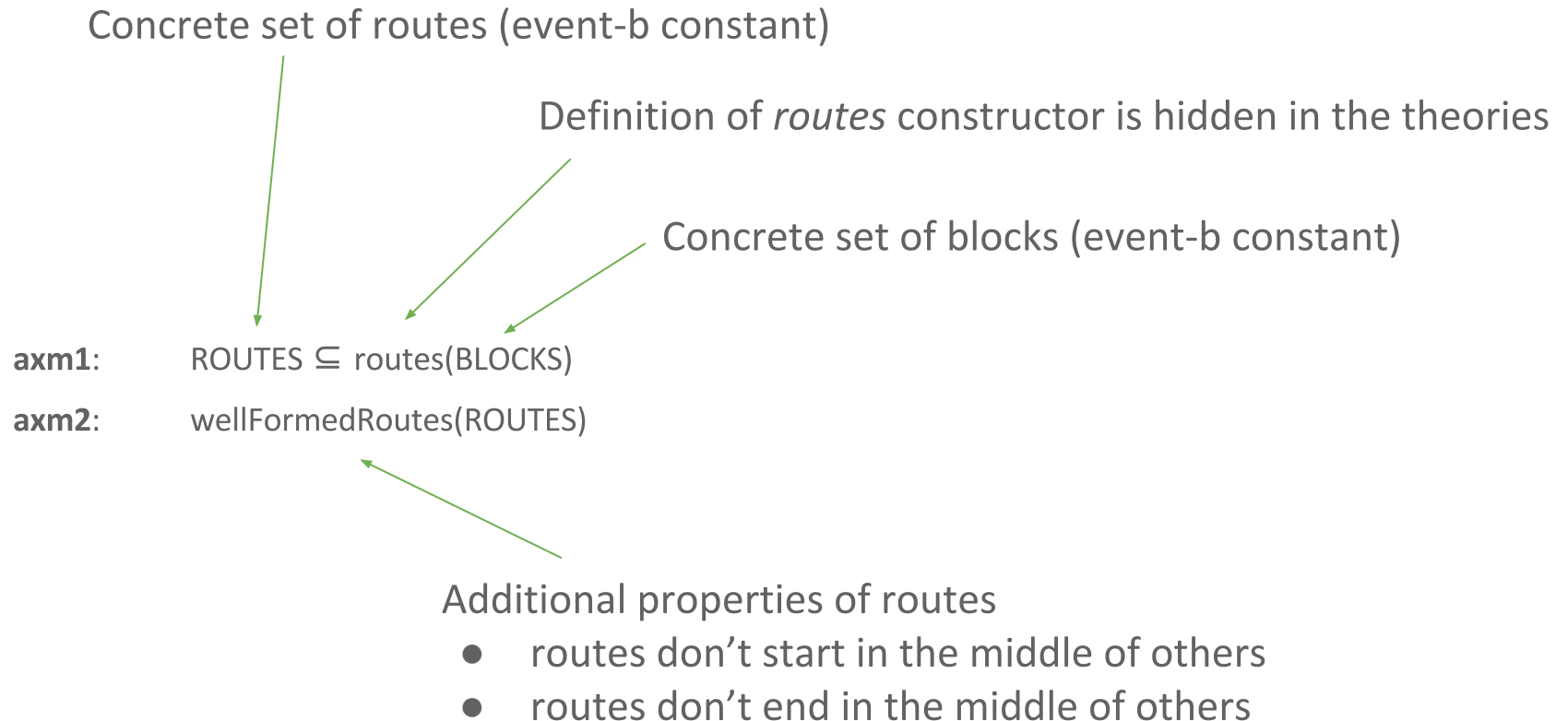
# Using Our Theories

**axm1:**       $\text{ROUTES} \subseteq \text{routes}(\text{BLOCKS})$

**axm2:**       $\text{wellFormedRoutes}(\text{ROUTES})$



# Using Our Theories



# The Famous Train Example First Machine

## Without Theories

```
route_reservation:
ANY r WHERE
  r  $\notin$  res_routes

  blocks_routes~[{r}]  $\cap$  res_blocks =  $\emptyset$ 

THEN
  res_routes  $\models$  res_routes  $\cup$  {r}
  resbl_resrt  $\models$  resbl_resrt  $\cup$  (blocks_routes  $\triangleright$  {r})
  res_blocks  $\models$  res_blocks  $\cup$  blocks_routes~[{r}]

END
```

## Using Our Theories

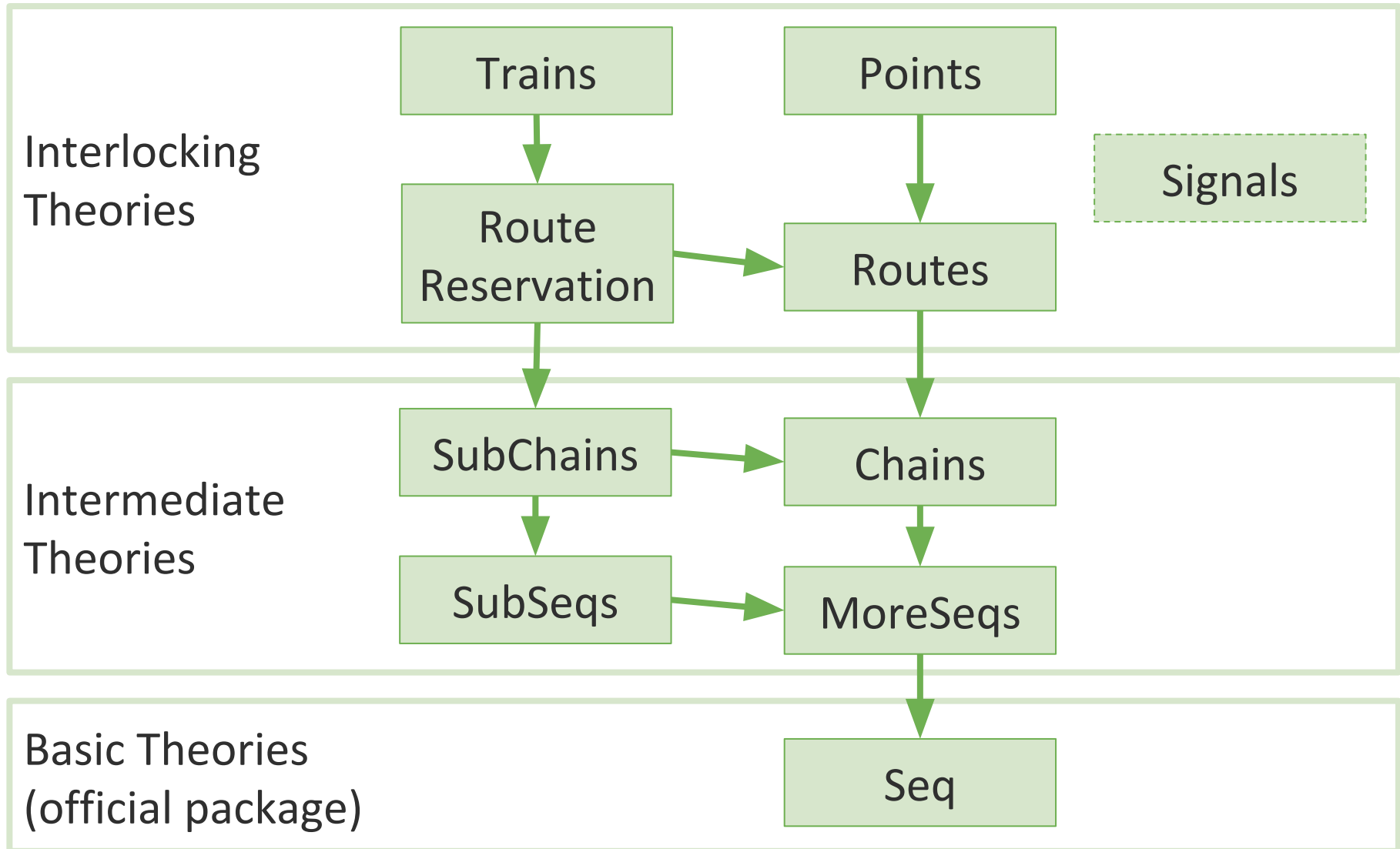
```
route_reservation:
ANY r WHERE
  r  $\in$  ROUTES
   $\neg$  isReserved(r, res_routes)
  noReservedBlocks(r, res_routes)

THEN
  res_routes  $\models$  res_routes  $\cup$  completeRes(r)

END
```

# Defining Interlocking Theories

# Interlocking Theories Dependencies



Railway network is made of chains of blocks.

$$\{ c \mid c \in \text{seq}(S) \wedge c \sim \in T \sqsubseteq \mathbb{Z} \}$$

equivalent to iseq  
guarantees that there is no cycle

Operators:

- chains (constructor)
- chainSize
- emptyChain
- chainFst / Last / Tail
- chainPrev / Next
- chainPrepend / Append
- ...

Theorems:

- chainIsFinite
- chainIsMonotonic
- nextIsInRange
- chainTailIsChain
- chainPrependIsChain
- ...

Routes are chains of at least two blocks.

$$\{ c \mid c \in \text{chains}(\text{blocks}) \wedge \text{card}(c) > 1 \}$$

## Operators:

- routes (constructor)
- blcks
- routeLength
- routeFst / Last / Tail
- routePrev / Next
- starts / endsInTheMiddle
- wellFormedRoutes
- ...

## Theorems:

- routelsFinite
- routelsMonotonic
- routelsFunc
- routeDomain
- routelsChain
- ...

# Routes (How To Use It ?)

Routes are chains of at least two blocks.

$$\{ c \mid c \in \text{chains}(\text{blocks}) \wedge \text{card}(c) > 1 \}$$

- $r$  is a route
- $b$  is a block of  $r$
- first block of  $r$
- block after  $b$  on  $r$
- $r1$  starts in the middle of  $r2$
- $R$  is a set of routes
- no route in  $R$  ends in the middle of another

$r \in \text{routes}(\text{BLOCKS})$

$b \in \text{blcks}(r)$

$\text{routeFst}(r)$

$\text{routeNext}(\{b\}, r)$

$\text{startsInTheMiddle}(r1, r2)$

$R \subseteq \text{routes}(\text{BLOCKS})$

$\text{routesDontEndInTheMiddle}(R)$

Routes reservations and trains are subchains of routes.

$$\{ \text{sub} \mid \text{sub} \in \text{chains}(T) \wedge \text{subChain}(\text{sub}, c) \}$$

Operators:

- `subChain` (constructor)
- `subChains` (constructor)
- `frontSubChains` (constructor)
- `backSubChains` (constructor)

Theorems:

- `subChainIsChain`
- `frontSubChainIsSubChain`
- `backSubChainIsSubChain`
- `chainTailIsABackSubChain`
- `chainTail`
- ...



# Subchains (How To Use It ?)

Routes reservations and trains (routes occupation) are subchains of routes.

$$\{ \text{sub} \mid \text{sub} \in \text{chains}(T) \wedge \text{subChain}(\text{sub}, c) \}$$

- |                                 |                                |
|---------------------------------|--------------------------------|
| • r1 is a subchain of r2        | <code>subChain(r1, r2)</code>  |
| • set of subchains of r         | <code>subChains(r)</code>      |
| • set of “front subchains” of r | <code>frontSubChains(r)</code> |
| • set of “back subchains” of r  | <code>backSubChains(r)</code>  |

# Routes Reservations

A theory for maintaining the set of routes reservations.

$$\{ r \mapsto b \mid r \in \text{routes}(\text{blocks}) \wedge b \in \text{backSubChains}(r) \}$$

## Operators:

- allRouteRes (constructor)
- validRouteRes (constructor)
- onlyOneResByRoute
- compatibleRoutesOnly
- wellFormedRouteRes
- blocksResForThisRoute
- isReserved
- addToRouteRes
- freeResHeadBlock
- ...

## Theorems:

- routeReservationIsAFunction
- routeResAreBackSubChains
- addRouteResStillOnlyOne
- addRouteResStillCompatible
- onlyOneResByRouteTrans
- filterResUnion
- ...

# Routes Reservations (How To Use It ?)

A theory for maintaining the set of routes reservations.

$$\{ r \mapsto b \mid r \in \text{routes}(\text{blocks}) \wedge b \in \text{backSubChains}(r) \}$$

- the set of route reservations on the track  $\text{res} \in \text{validRouteRes}(\text{BLOCKS}, \text{ROUTES})$
- only one reservation can be made for each route  $\text{onlyOneResByRoute}(\text{res})$
- a block cannot be reserved twice  $\text{compatibleRoutesOnly}(\text{res})$

# Routes Reservations (How To Use It ?)

A theory for maintaining the set of routes reservations.

$$\{ r \mapsto b \mid r \in \text{routes}(\text{blocks}) \wedge b \in \text{backSubChains}(r) \}$$

$$r = \{b1; b2; \dots; bn\}$$

## Events

Initialisation

Reserve  $r$

Enter route; Front moves...

Back move

Back moves...

Free  $r$

## Route Reservations

$\{\}$

$\{ r \mapsto \{ b1; b2; \dots; bn \} \}$

$\{ r \mapsto \{ b1; b2; \dots; bn \} \}$

$\{ r \mapsto \{ b2; \dots; bn \} \}$

$\{ r \mapsto \{\} \}$

$\{\}$

# Reducing the Proof Effort ?

Conclusion

# Current Status

Theorems defined = proof factorized.

Manual proof is easier, sometimes even trivial.

But only 40% POs automatically discharged in our theories and models.

Defining theories does not naturally simplify the proof.

# The Right Theorems... (future work)

How to define the right theorems ?

Suggestion:

1. define more theorems... a lot of them !
2. generalization of the theorems
3. simplification of the theorems

# Proof Strategies? (future work)

Theorems not automatically applied: use strategies ?

How to define them ?



# Discussion (1/2)

- Infix predicates ?
  - $\text{startsInTheMiddle}(r1, r2) \Rightarrow r1 \text{ startsInTheMiddle } r2$
- Type inference ?
  - $r \in \text{ROUTES}$ 
    - $\neg \text{isReserved}(r, \text{res\_routes})$
- Local theories ? Using symbols that are local for a given project.
  - $\neg \text{isReserved}(r, \text{res\_routes})$
- Assignment operators ?
  - $\text{res\_routes} := \text{res\_routes} \cup \text{completeRes}(r)$

- Theory instantiation?
  - ex: Instantiate the Routes theory with the constant BLOCKS
- What about a Rodin plug-in for generating *Domain Specific Platforms* (DSP) ?
  1. Define the DSL using the Theory plug-in
  2. Validate the DSL with signaling engineers
  3. Generate the DSP
  4. Let signaling engineers model their system



# Thanks

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