A (very) small experiment in Event-B rippling

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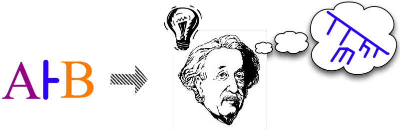
Talk outline

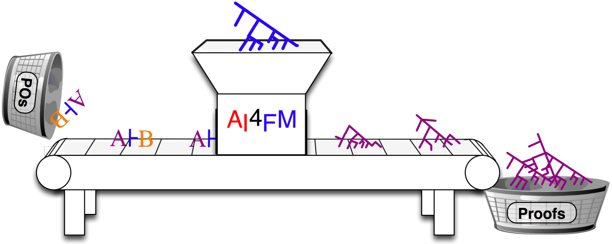
- "Abrial's MMPE rule" : $n * x/100 * f * p * 20$
  - $100,000$ loc $\rightsquigarrow$ 3-12 Man Months of Proof Effort
- BUT, $\exists$ families of Event-B UPOs based on proof strategy
  - AI4FM tries to explore such families to increase automation.
- This requires high-level proof strategies
  - rippling is an example of a high level proof strategy
  - implemented in Isaplanner.
- In this talk we will:
  - give an overview of the AI4FM project
  - describe a simple Event-B experiments with rippling/Isaplanner
  - beg for help!

AI4FM overview

- The user manually proves one exemplar proof.



- The theorem prover uses the additional information from the exemplar proof to discharge "similar" proofs:



The project

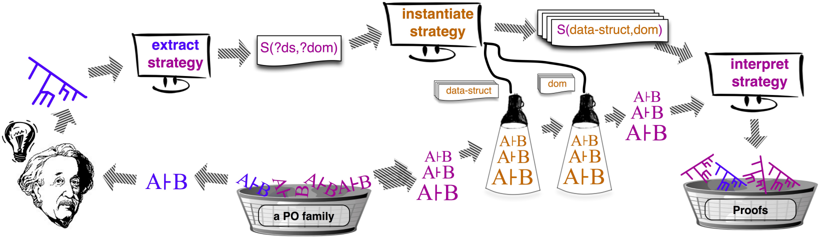
- 4 years UK EPSRC funding – started 1 April 2010
  - EP/H024050/1, EP/H024204/1 and EP/H023852/1.
- The team
  - Newcastle
    - Cliff Jones, Leo Freitas, Andrius Velykis
  - Edinburgh
    - Alan Bundy, Gudmund Grov, Yuhui Lin
  - Heriot-Watt
    - Andrew Ireland
  - Southampton
    - Michael Butler.

Nature of Event-B POs/proofs

- Rarely deep.
  - we are *not* trying to prove real math theorems!
- Complexity reduced by layering abstractions.
- Lots of POs
  - ... which can often be grouped into "(proof) families".
- Lots of detail (on larger examples)

The AI4FM process

- (Somehow) classify POs into families.
- Require the expert user to prove 1 PO manually/interactively
  - preferable the "simplest".
- This extra information is used to discharge rest of families
- Requires abstracting proof into a higher-level strategy
  - ... and we must design a strategy language to capture this.



Towards a strategy language

- The strategy language needs to be robust to cope with changes.
- A strategy may describe a sequence of intermediate lemmas that could be spawned:

  prove lemmas $L_1, L_2, \cdots, L_n$ s.t. GOAL follows.
- Abstract over many "dimensions"
  - data-structure, domain, etc.
- Include notions like generalisation and lemma discovery.
- Rippling provides evidence for a high-level strategy language.

Rippling (in a hurry)

- Proof plans: high-level description of proofs
  - captures common patters of reasoning
- Rippling: a proof plan, which
  - works when one of the givens can be embedded in the goal (e.g inductive step cases)
    - for example in an Event-B INV type PO:

$$com = Cls \lhd (cl; nm) \vdash com = Cls \lhd (\boxed{(cl \lhd\!\!\!- \{s \mapsto (sze \mapsto [])\})}^{\uparrow}; nm)$$

  - annotations to guide rewriting (towards given)
    - language of **wave-fronts** and **skeletons**
  - (direction) guarantees termination

Rippling illustration (step case of $t@(Y@Z) = (t@Y)@Z$)

Rewrite rules:

$$\boxed{H\#\,T}^{\uparrow}@L \Rightarrow \boxed{H\#\,T@L}^{\uparrow} \qquad (1)$$

$$\boxed{X_1\#\,X_2}^{\uparrow} = \boxed{Y_1\#\,Y_2}^{\uparrow} \Rightarrow \boxed{X_1 = Y_1 \wedge X_2 = Y_2}^{\uparrow} \qquad (2)$$

Rippling proof:

$$\begin{array}{ll}
t\ @\ (Y\ @\ Z) = (t\ @\ Y)\ @\ Z & \text{IH} \\
\boxed{h\#\,t}^{\uparrow}@(Y@Z) = (\boxed{h\#\,t}^{\uparrow}@Y)@Z & \text{apply (1) 2 times} \\
\boxed{h\#\,t@(Y@Z)}^{\uparrow} = \boxed{h\#(t@Y)}^{\uparrow}@Z & \text{apply (1)} \\
\boxed{h\#\,t@(Y@Z)}^{\uparrow} = \boxed{h\#(t@Y)@Z}^{\uparrow} & \text{apply (2)} \\
\boxed{h = h \wedge t@(Y@Z) = (t@Y)@Z}^{\uparrow} & \text{apply IH}
\end{array}$$

IsaPlanner

- Isaplanner is a proof planner built on top of Isabelle
  - enables use of existing Isabelle automation
  - soundness ensured by Isabelle.
- Reasoning techniques which generates proof plans
  - lazy search over possible ways to apply technique
- Implemented in the ML language
  - language also used to write new reasoning techniques
  - provides many ML functions to aid developing new techniques
- Rippling technique encoded (among others)

The experiment

- Goal is to check how rippling works in an Event-B setting
- Longer term we hope to see how easy it is to encode new techniques/patterns in Isaplanner.
- Set representation in Isabelle/HOL used
  - ... and slightly extended
- Modelled an example system in Rodin
  - translated POs into Isabelle/HOL representation
- Addressed INV type POs

Representing Event-B POs in Isabelle/HOL

- Event-B POs (+ theory) must be represented in Isabelle/HOL
- Uses Isabelle/HOL's set theory
- Extended with "Event-B operators"
  - e.g. $\lhd$, $\rhd$, $⩤$, $⩥$, ...
- Functions as relations (HOL functions are total)
  - $\rightarrow$, $\nrightarrow$, $\twoheadrightarrow$, ... defined
  - function application uses definite description operator (i.e. ι)
- Proof rules (theorems) derived by need
- Drawbacks
  - Most general type for functions
    - little milage from HOL type checking
  - WDs ignored
  - Ignores Event-B proof rules

The system: a telephone system

- Based on Z model by Woodcock
- The variables
  - $call \in Subs \nrightarrow (Status \leftrightarrow Subs)$
  - $connected \in Subs \nrightarrow Subs$
  - st and num projections on $(Status \leftrightarrow Subs)$
  - $Free \subseteq Subs$
- Invariants

$$
\begin{aligned}
inv1: &\quad Callers = dom((call; st) \rhd Connected) \\
inv2: &\quad connected = Callers \lhd (call; num)
\end{aligned}
$$

- where $Connected \subset Status$ for "connected calls".

The system: a telephone system (events)

- Event's to lift handle, dial, answer, etc..
- We will only discuss two events
  - **EVENT** *LiftFree* $\widehat{=}$
    ANY s **WHERE** $s \in Free$
    THEN $Free := Free \setminus \{s\}$
    $call(s) := (seize \mapsto empty)$
    $\ldots$
  - **EVENT** *LiftSuspended* $\widehat{=}$
    ANY s **WHERE** $(s \mapsto suspended) \in connected^{-1}; call; st$
    THEN $call(connected^{-1}(s)) := (speech \mapsto s)$
    $\ldots$

PO LiftFree/inv1/INV

- All required rewrite (wave) rules are existing
- Proof strategy
  - Rippling followed by application of IH (inv1)
  - Then manually discharge reminding (non-rippling) goals
    - i.e. the conditions from conditional rewrite rules
- PO LiftFree/inv1/INV:

$$\mathit{Callers} = \mathit{dom}(\mathit{call}; \mathit{st} \rhd \mathit{Connected}), s \in \mathit{Free} \quad \vdash$$
$$\mathit{Callers} = \mathit{dom}((\mathit{call} \mathbin{\lhd\!\!-} \{s \mapsto (\mathit{seize} \mapsto \mathit{empty})\}); \mathit{st} \rhd \mathit{Connected})$$

- (in one branch) gives the following two (provable) sub-goals

$$s \notin \mathit{dom}(\mathit{call}) \qquad ((s \mapsto (\mathit{seize} \mapsto [])); \mathit{st}) \rhd \mathit{Connected} = \{\}$$

- note there are also several (sometimes unprovable) branches

PO LiftFree/inv2/INV

- PO LiftFree/inv2/INV:

$$connected = Callers \lhd (call ; num), s \in Free \vdash$$
$$connected = Callers \lhd ((call \Lleftarrow \{s \mapsto (seize \mapsto empty)\}); num)$$

- (in one branch) gives the following two (provable) sub-goals

$$s \notin dom(call) \qquad (Callers \lhd \{(s \mapsto (seize \mapsto []))\}; num) = \{\}$$

- The "idea" is similar to LiftFree/inv1/INV
  - "ripple out" to "isolate" the new part – and show it is $\{\}$
  - BUT the rules used, i.e. the proof, differs (e.g. $\rhd$ vs. $\lhd$)
  - AND one of the sub-goals is harder to prove

LiftSuspended POs

- PO LiftSuspended/inv1/INV

$$Callers = dom(call; st \rhd Connected),$$
$$(s \mapsto suspended) \in connected^{-1}; call; st \vdash$$
$$Callers = dom((call \Leftarrow \{connected^{-1}(s) \mapsto (speech \mapsto s)\}); st \rhd Connected)$$

- PO LiftSuspended/inv2/INV

$$connected = Callers \lhd (call; num),$$
$$(s \mapsto suspended) \in connected^{-1}; call; st \vdash$$
$$connected = Callers \lhd ((call \Leftarrow connected^{-1}(s) \mapsto (speech \mapsto s)); num)$$

- Previous approach: s was not in *Callers*, and is still not
- Here, the "dual" is true:
  - $connected^{-1}(s)$ was in *Callers* and still is.
- Requires "theory" properties of *num*, *st*, *call* and *connected*

Some observations: difference with existing rippling work

- Hard to make any conclusions from a small case study
- Rippling seems promising for INV POs
- It manages to reduce the PO to a smaller, simpler goal – however:
  - much more conditional rewriting than other domains
  - .. and WD (which will add further conditions) ignored
  - not clear how to discharge these (non-rippling) goal
- Still lots of room for improvement:
  - several branches – use counter-example finders to filter out most obvious ones
  - better use of existing techniques (simp, blast, etc)
  - productive use of failure.

Productive use of failure

- One advantage of rippling is Ireland's proof critics.
- Particular failures (or partial matches) of rippling triggers certain "exception cases"
  - e.g. a missing lemma is speculated, or the conjecture is generalised.
- More robust than lower-level tactics.
- These could be applied in the Event-B setting
  - but not clear how to prove discovered/generalised lemmas
    - induction+rippling traditionally used
    - ... this may not be the case in Event-B
  - we may need to discover new proof strategies here
  - ... or learn them from a given exemplar proof ...

Rippling experiment vs. AI4FM agenda

- AI4FM is about learning strategies from an exemplar proof
- Rippling is an existing strategy
- Maybe a general strategy for INV is to use rippling
- and the special purpose strategies learned from exemplar proofs are used to
  - discharge sub-goals from conditional rewrite rules in target proofs
  - .. and (internally) of source proof, use proof of one such sub-goal to discharge other
  - prove discovered lemmas.

The AI4FM plan

- Analyse a lot of
  - POs
    - preferably from real-world applications
  - ... their (expert-provided) proofs attempts
  - and analysing "families".
- Based on analysis, *develop a strategy language*
  - more examples $\Rightarrow$ more robustness
  - thus, the need for examples!
  - we expect an iterative development of the language.
- Provide tool support
  - to extract a strategy out of an exemplar proof
  - to interpret strategies to discharge "similar POs".

We need your help (so we can help you)!

- We have experimented with our own little case-study.
- Steve Wright have already volunteered to participate!
- You can help by providing case-studies with (non-trivial) POs:

- ... industrial-sized preferably
- ideally, with (given) families of proofs
- even better with with proof history (including dead-ends)
- the full proof process can tell us more than the finished article

- Examples from difference sources will increase robustness
- ... which will (hopefully) give you better proof automation!

Summary

- AI4FM is a new project trying to learn and recycle proof strategies.
- Rippling is a high-level proof strategy
- We have shown promising results when rippling Event-B POs
  - a simple example
  - user-interaction still required
- (Your?) relevant example(s) will be of great help!

Thank you!

AI4FM

http://www.ai4fm.org