

Examples of using the Instantiation Plug-in

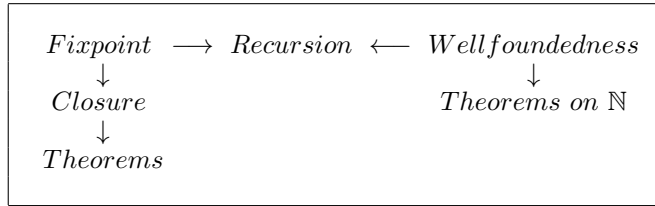
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In a companion paper [1], Guillaume Verdier and Laurent Voisin presented a new approach to genericity in the Rodin toolset: this approach is made practical by means of an Instantiation Plug-in. In the present short paper¹ we propose some examples of using this new approach. Note that we constructed more examples: we only present here the most important ones. These examples are preliminary as the plug-in is still under development as stated in [1]. The key to this presentation is to show how such examples can be structured using the Instantiation Plug-in.

Two basic examples are independent: Fixpoint and Wellfoundedness. Other examples depend directly or indirectly of them. This is indicated in the following diagram.



1 Fixpoint

Given a set S and a set function h built on S : $h \in \mathbb{P}(S) \rightarrow \mathbb{P}(S)$, a fixpoint of h is a subset $\text{fix}(h)$ of S such that $\text{fix}(h) = h(\text{fix}(h))$. Here is a proposal:

$$\text{fix}(h) \hat{=} \text{inter}(\{s \mid s \subseteq S \wedge h(s) \subseteq s\})$$

Assuming that the function h is monotone; we have (Tarski)

$$(\forall a, b. a \subseteq b \Rightarrow h(a) \subseteq h(b)) \Rightarrow \text{fix}(h) = h(\text{fix}(h))$$

Moreover $\text{fix}(h)$ is the least fixpoint:

$$\forall t. t = h(t) \Rightarrow \text{fix}(h) \subseteq t$$

2 Closure

Given a set S and a relation r built on S : $r \in S \leftrightarrow S$, the closure(r) is defined to be the following fixpoint:

$$\text{closure}(r) = \text{fix}(\lambda s. s \in S \leftrightarrow S \mid r \cup (s; r))$$

Since we use the fixpoint operator, we have to instantiate (adapt) its definition as provided within the corresponding section. This is exactly what is allowed by the Instantiation Plug-in.

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3 Closure Theorems

We now instantiate the closure definition and prove the following theorems:

$$\text{closure}(r) = \text{fix}(\lambda s \cdot s \in S \leftrightarrow S \mid r \cup (s ; r))$$

$$\text{closure}(r^{-1}) = (\text{closure}(r))^{-1}$$

4 Wellfoundedness

We are given a set S and a binary relation r built on S : $r \in S \leftrightarrow S$. If for any x belonging to the range of r , we follow r^{-1} and reach a point which is not in the range of r after a *finite* travel, the relation r is said to be well-founded: $\text{wf}(r)$. It can be given the following formal definition:

$$\text{wf}(r) \hat{=} \forall p \cdot p \subseteq S \wedge p \subseteq r[p] \Rightarrow p = \emptyset$$

It can be proved that we have an induction rule for a set with a well-founded relation. Here is the corresponding formal definition:

$$\forall q \cdot q \subseteq S \wedge (\forall x \cdot x \in S \wedge r^{-1}[\{x\}] \subseteq q \Rightarrow x \in q) \Rightarrow S \subseteq q$$

5 Theorems on Natural Numbers

We now instantiate the set S of previous section to the set \mathbb{N} of natural numbers. It can be proved that the relation "+1" on \mathbb{N} is well-founded. As a consequence, we can deduce the classical induction rule for \mathbb{N}

6 Recursion

We are given a set S and a well-founded relation r built on S . We are given another set B and a function g defined as follows: $g \in (S \times (S \leftrightarrow B)) \leftrightarrow B$. Then there exists a unique total function f from S to B with the following property:

$$\forall x \cdot x \in S \Rightarrow f(x) = g(x \mapsto r^{-1}[\{x\}] \triangleleft f)$$

This function is defined by means of a fixpoint. We have thus to instantiate the definition of wellfoundedness and that of fixpoints defined in earlier sections.

7 Conclusion

All proofs alluded in this paper were successfully performed using the Instantiation Plug-in and the Rodin Tool.

References

1. Guillaume Verdier and Laurent Voisin: *Context Instantiation Plug-in: a new approach to genericity in Rodin*